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ABSTRACT

This document begins with a discussion of the author's approach to instruction in a mathematics laboratory. This discussion includes an enumeration of types of desirable or necessary equipment and advice on the management of a laboratory. The author examines issues related to achievement and readiness for more traditional school experiences in later grades, and offers points from his own educational philosophy. The major portion of the document is devoted to detailed descriptions of activities or series of activities which have been used successfully. Activity topics are quite diverse and include numeration systems, number lines, measurement, geoboard activities, map making, geometric solids, and others. The document concludes with reproductions of activity cards used by the author. (SD)

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Learning to Think in a Math Lab

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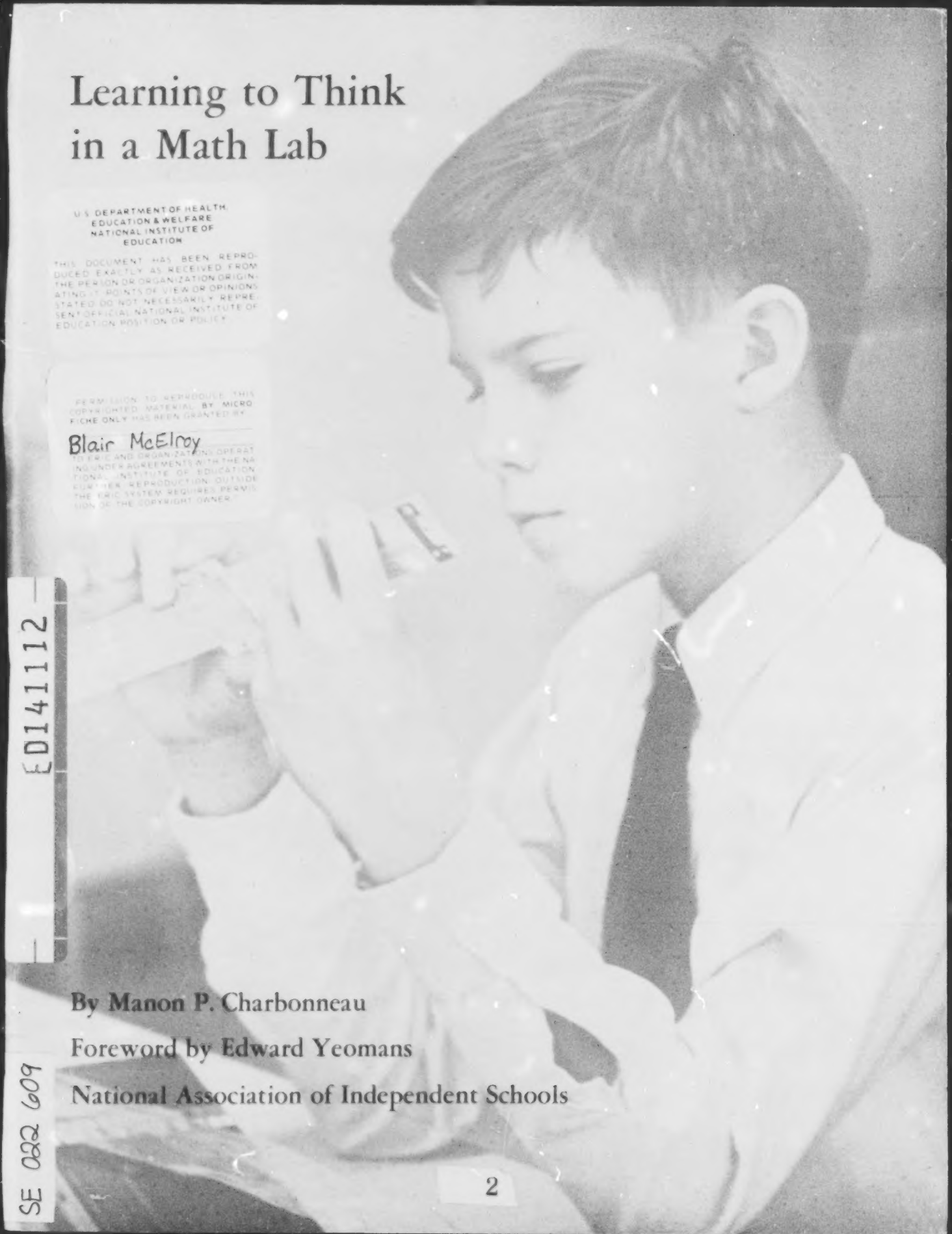

By Manon P. Charbonneau

Foreword by Edward Yeomans

National Association of Independent Schools

LEARNING TO THINK
IN A MATH LAB

by Manon P. Charbonneau

Colorado Academy, Denver, Colorado

Braitmayer Fellow, 1969 — 1970

Foreword by Edward Yeomans

April 1971

National Association of Independent Schools

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ACKNOWLEDGMENTS

Throughout my years of teaching, both at Hawken School in Cleveland and Colorado Academy, I have had the privilege of working and studying with some of the most outstanding individuals in the field of education. Their interest in and sensitivity to young children, and their dedication to finding out how children learn--"what makes them tick"--as well as to the discipline of expanding the scope of mathematics of those of us who are learning, have given me the confidence to put these observations together in manuscript form. My grateful thanks to: David Armington, Allen Banbury, Edith Biggs, Gordon Clem, Richard Ward Day, Dr. Patricia Davidson, Dr. Robert Davis, Dr. Caleb Gattegno, Dr. David Hawkins, John Holt, William P. Hull, Roy Illsley, Madison Judson, Anthony Kallet, Dr. David Page, Lore Rasmussen, Leonard Sealey, Ramon Steinen, Frank S. Trautman, John Trivett.

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All photographs were taken by Dave Buresh, Denver, Colorado.

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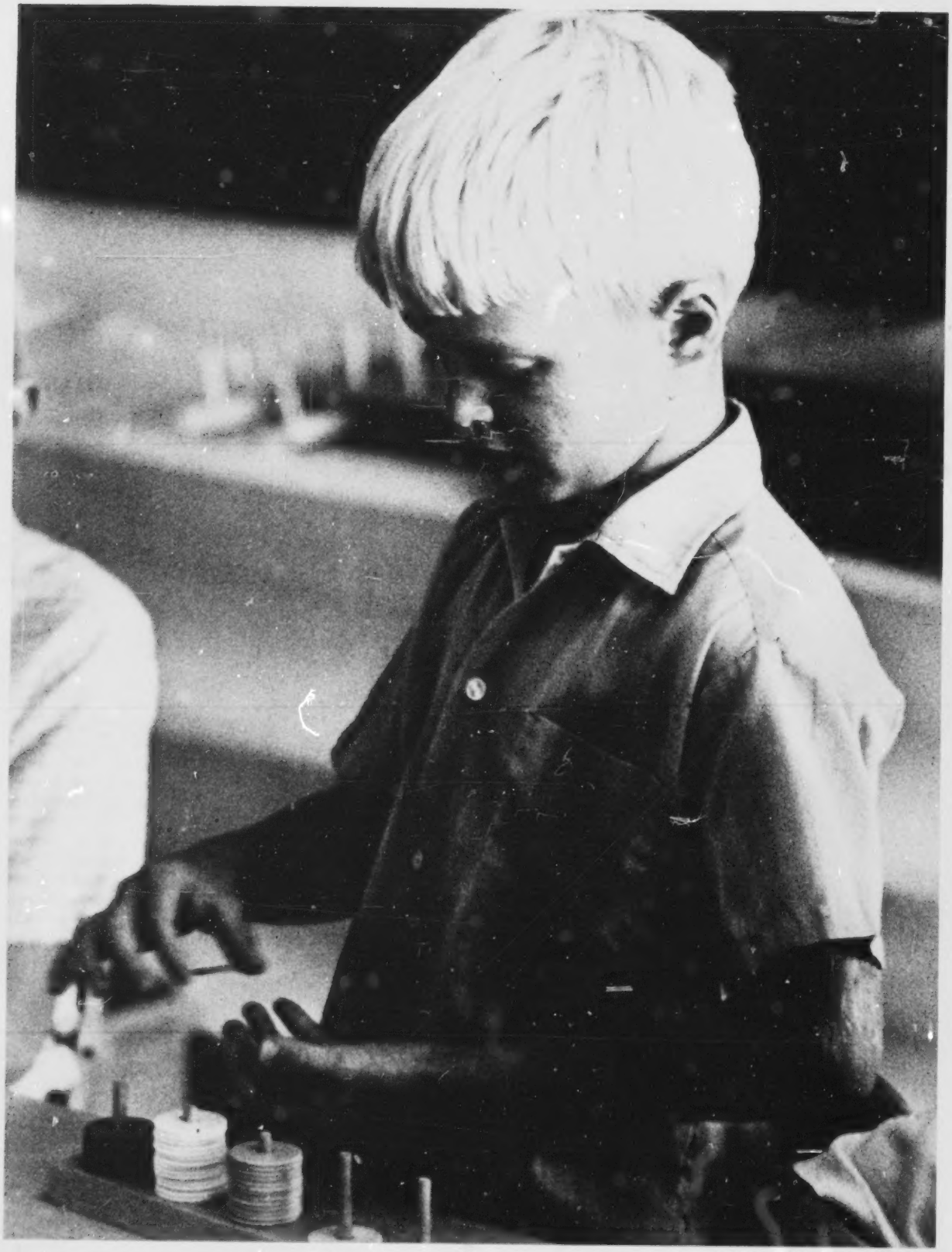
I dedicate this book to my family, whose patience with my erratic hours and incoherent conversation for a year made this project a reality.

M.P.C.



FOREWORD

The literature of education is notorious for its attempts to measure the unmeasurable, to define the indefinable, and to describe the indescribable. Educators, instead of admitting that learning is an act of spirit as well as of mind, and that the connection between teaching and learning is often tenuous, try to adapt the terms and methods of the scientist to a thoroughly unscientific process. The pedagese that results often does more to obscure the truth than to illuminate it.

It is refreshing, therefore, to come face-to-face with an artist-teacher, whether through the written word, the moving picture, or best of all, in flesh and blood. The distinguishing characteristic of such a teacher is that he sees himself primarily as a facilitator of learning, a catalytic agent in a process that could be likened to photosynthesis. He supplies some of the elements from his own learning, from his experience with life and people, and from the environment he creates. But his students supply other elements, equally necessary, from their knowledge of life and of people and their innate sensitivity to truth, orderliness, and humane feelings. The art of teaching lies in the arranging, by the teacher, of all of these elements in such proportion that a synthesis of experience takes place — as much within him as within his students.

This is very different from the sort of teaching, still common in school and college, which invites learners to come before the Authority and Receive; and if something is wrong with the reception, if there is static, or if the channels are overloaded, the student fails. The agonies which engulf western society must be caused, in part at least, by this kind of teaching because practically everyone has spent parts of his most impressionable years in school, and most of the leaders have gone on to colleges and graduate schools.

Manon Charbonneau is an artist-teacher in the tradition of Sylvia Ashton-Warner and Sybil Marshall. Her math lab is the environment within which she, her students, visitors, other teachers, other students, interact with one another. Mathematics is one of the bonds they have in common, but so are intellectual engagement, and problem-solving, and appreciation of individual differences, and aesthetic pleasure in rhythm and design, and on and on into all the ramifications of life as it may touch those in the act of living it. That her students can also pass objective tests in mathematics is what should be expected of anyone who has lived intelligently in a world of quantity - and is not particularly important. What is important is the kind of person who emerges from these experiences: the ways in which he perceives reality, his concerns about moral values, his awareness of himself and others in society.

Within this pleasantly discursive record there are several major sign posts. One says, in effect: "Play and invention can be educational." Another says: "Listen to the students, for they may have an even better way of doing it." A third says: "You can learn as much from their mistakes as from their successes." And again: "The ways in which you mark and judge students' work have much to do with their learning." And finally: "First the reality must be understood: then the symbols - the abstractions - will have meaning."

These are the penetrating aims of a teacher who values intellectual discipline and structure of thought, yet who approaches children light-heartedly, with appreciation of their qualities as people. The "Method" used — the sequences, the lesson cards - are, of course, her own as any method must be to be valid equally for students and teacher. No text, no packaged curriculum, however embellished with extraneous aids to motivation, can ever equal the satisfaction of a teacher's own planning as it grows from work with children.

As a Braitmayer Fellow for the year 1969-70, Mrs. Charbonneau sorted out her notes and materials, collected over a period of twelve years of teaching, for this report. She is at present a member of the faculty of Colorado Academy where she has been for five years. Before that she taught for seven years at Hawken School. She has been a member of the staff of the Madison Project and of the NAIS Workshops in Mathematics and the Integrated Day, and has also run workshops for individual schools and school districts in various parts of the country.

NAIS is grateful to the Braitmayer Foundation for making possible the time required to prepare this document, and to Mrs. Charbonneau for permission to publish it.

EDWARD YEOMANS
Associate Director of Academic Services
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A mathematics laboratory is a learning environment more than anything else. It is a place full of "things": games to be played, tools and equipment for experimentation, blocks for building beautiful structures, patterns, concepts; boxes and bottles, shapes of all kinds; scales and balances, rulers, measuring devices in standard and nonstandard units; papers and pencils and crayons and marker pens; spools, little toy cars, trains, dried beans, peas, rice, salt, sugar, coffee, cups, spoons – to name just a few. It is a place where imagination and creative ideas are born and grow, where thoughts and feelings can become certainties, where many young minds decide that mathematics is certainly more than practicing "sums" that teacher assigns. It is a place where problems are created and ways to solve them are thought up by the individuals involved. It is a place where one can learn just about all he wants to know about numbers, and what happens to them "if" . . . and it can be learned slowly, using lots of "stuff" to prove or disprove a point, or it can be found quickly because that eager curiosity moved like lightning that day.

There is so much that can be said for a mathematics laboratory for the use of young children of all ages, really, that it is difficult to be definitive in one's approach to setting one up. In this book I am primarily concerned with the upper elementary grades – the nine-, ten-, and eleven-year-olds. Children vary from school to school, so that one must be acutely aware of and sensitive to the needs of the individual children with whom one will be working when stocking a lab. There are some things I personally could not do without, others that are desirable but not necessary, and then there are the frills – those special tools or games or books or blocks that would be great if only they did not cost so much; but many, many things can be put into a lab which cost little or nothing but the time involved in scrounging them up, or making them in some instances.

I strongly suggest measuring tools of all types: tin measuring cups, measuring spoons, quarts, pints, half pints, half gallons, gallons (milk containers that can easily be gathered and replaced with use), all sizes of boxes, bottles, containers that are cylinders or odd shapes, styrofoam shapes of all sizes, yardsticks, meter sticks, foot rulers, trundle wheels in yards and meters and odd sizes, tongue depressors, sticks of various lengths, floor tiles (9" by 9" and 12" by 12", etc.), scales of all kinds, gram weights and ounce weights, marbles – which can be used for weighing very effectively. Bottles and jars can come from any kitchen or medicine cabinet. Never throw away odd-length sticks; these can be sanded down and used for measuring with nonstandard units. Tongue depressors are ideal for many things, and doctor fathers are often willing to donate a box or two of them! String of all lengths and weights, yarn, rope – all can be easily collected, and once you start looking, boxes of all assorted shapes and sizes start popping up. These are great for volume and surface area problems. Quart plastic containers with rice, dried cereal, macaroni, dried beans, dried peas, coffee, tea, sugar, salt, flour, sand, popcorn are all great things to weigh and measure and compare. Standard cups, medicine cups (small paper ones), plastic spoons or scoops for getting the stuff out of the containers, or for use in weighing small quantities are just ideal. (I have had children balance one cup of rice against plastic spoons or marbles instead of weights – and often it's fun just to see how much tea will balance one half cup of rice etc.) Children can invent many, many ways themselves to tell you "how much" a thing weighs, and when getting started with this sort of thing, nonstandard units are more interesting and fun than standard ones. And, when you find out for yourself, with the gram weights, just how many grams equal one ounce, it sticks – far longer than going to a table of reference in a book – and it's more fun!

One tremendous motivator in the area of measurement is a beautiful full-color chart I received gratis through the mail from the Ford Company on the history of measurement. It tells, quite simply and graphically, the origin of such things as the foot, the cubit, the rod, the fathom, the hand, and goes right on up to such fantastic measuring units of today as engstroms. One group of ten-year-olds I had became so fascinated with the chart that they made their own "feet," "hands," cubits, yards, fathoms, rods, etc., and proceeded to measure everything in sight with their own units. They even came to the conclusion that standardized units of measure are a very real necessity, or how else can you be sure of what you are getting? All I did was put the chart on the wall – everything else came from them. If I had suggested the study, I doubt that any of the boys would have done more than grunt or snort their disapproval. What they ended up doing took far more work and care than anything I could have thought up for them – and they enjoyed it much more.

As far as blocks go, I am very fortunate. I have in my lab Cuisenaire rods (which I consider indispensable), Dienes algebraic experience materials and multi-base arithmetic blocks, Attribute blocks, pattern blocks, and geometry blocks, all from EDC.[1] I also have a box full of inch cubes and two-inch cubes just cut and sanded from plain lumber.

Games include dominoes, triminoes, fraction dominoes, Vector, Quinto, Twixt, Yah-ztee, Tuf, Stocks and Bonds, Milles Bornes, Equations, Hex, Hi-Q, and a variety of puzzles that are nameless as far as the market goes. There are lots of three-dimensional wooden puzzles

[1] Education Development Center, 55 Chapel Street, Newton, Massachusetts 02160

for taking apart and putting back together, Insanity blocks, the Tower of Hanoi, Kalah – a nifty marble game of "count and capture." We even have Scrabble and Spill and Spell – learning doesn't stop with just numerical games!

More things I could not do without are maps of all kinds: railroad systems maps, airlines systems maps, road maps, topographic sheets, world maps, political maps, underwater geographical maps, weather maps, and some Mercator projections of the world, as well as globes – both political and topographic. With the use of string or yarn and a ruler, the children can find distances both "as the crow flies" (airmail) and via roads and railroads and rivers, etc. A favorite game is to bring letters to school from all over the world and find out how far these have traveled to reach the addressees. And half the fun is finding the right map for the job!

Boxes full of compasses (drawing), protractors, simple cutting tools, all kinds of paper and cardboard can lead to scale drawings and then on to scale models, eventually in balsa wood. I have one young lad who designs the most beautiful sailing ships to scale, and then proceeds to color and describe them in detail – math, art, and English, all in one lesson. He has even produced some spectacular wooden models – and NOT from a kit, either!

Paper in a math lab is vital – plain typing paper, large sheets of colored paper, rolls of newsprint or large sheets of it, but above all, graph paper; one-inch squares, one half-inch squares, one quarter-inch squares, five squares to the inch, decimal graph paper, centimeter graph paper, large sheets, small sheets – we buy the enormous sheets and cut them down into various sizes. And adding-machine tape by the case – there is a multitude of uses for adding-machine tape. Something I would like to have, but do not, is a supply of very large sheets of centimeter graph paper for recording problems done with Cuisenaire rods. So far, I have found the small 8-1/2" by 11" size, but nothing larger – and there are times when the big sheets would be very desirable. Cardboard in all weights and thicknesses and sizes is very handy. Mat board is especially useful for scale models – it cuts easily with Exacto knives. Dot paper made for the 25-nail geoboard is also very important.

Geoboards are one of the key materials in the math lab. They could be treated as a subject by themselves! Their value as an exploratory device, as well as for more directed learning, cannot be overemphasized.

Miscellaneous items that come to mind which are very helpful and which the children find interesting to work with: Japanese and Chinese abacuses (the soroban and the swan pan), boards of various lengths (10, 12, 15 feet and 12 to 14 inches wide) for doing rate problems on an inclined plane, all kinds of little cars and trucks (Corgi cars, giant matchbox toys, etc.) and, ideally, an electric train! Problems involving rate, time, and distance mean much more when they can be worked with real "things." And the generalizations can then be more easily transferred to problems found in textbooks. Dice, magic markers, ball point pens, colored pencils, crayons, masking tape, scotch tape, colored tape, stop watches are a few more items that we make tremendous use of during the year.

"Things" come to mind as you work with the children and they work in the lab situation. Lots of things can be scrounged; little cars and trucks, milk cartons, plastic ice cream containers, empty spools, boxes and cartons and bottles. A simple letter to parents asking for cast-offs and throw-outs can bring in loads of materials.

DO NOT FEEL THAT YOU MUST START WITH A LOADED LABORATORY OF MATERIALS! We started with three bookshelves of games and puzzles, and over the course of four years have built up a very extensive mathematics laboratory. I would urge anyone starting in this situation to begin slowly, with a little bit of equipment, for short periods a day, and gradually build up into something which crosses hours, disciplines, and teachers. In making suggestions for stocking a lab, I cannot say strongly enough that you must meet the needs of the children with whom you are working, and as you work with them, ideas for materials and what to do with them constantly keep cropping up. What I have done here is give you a partial list of what is NOW in our lab at Colorado Academy for the elementary school, all of which is used constantly by the various age levels. I guess one criterion I use in deciding if a certain piece of equipment or game is right for our lab is its direct usability by the children. If whatever it is takes a lot of teacher direction or close supervision – in other words, something that the children cannot handle and use by themselves – then it is of very little value to me. I clearly remember Dr. Robert Davis, director of the Madison Project, saying to a child in a lab situation, "Play with it a while and see what it does, then we can talk about other things to do with it." Children as a whole are very creative about the things they work with, and the less you show them, the more they will invent ways to use them or things to do with them. Of course, they can think up destructive ways to use things too. We have two distinct rules in our lab: (1) YOU DO NOT BREAK OR DAMAGE ANYTHING YOU ARE WORKING WITH! and (2) NEVER INTERRUPT ANYONE ELSE WHO IS WORKING! Those two rules will cover almost any disturbing situation that can arise and spoil the lab set-up.

Before going any further with what to do in a lab and how to do it, let me tell you something about myself. I do not want you to think that I have an extensive formal background in mathematics, with degrees to go with it. How I wish I did, or at least had the time to take some more

courses in formal mathematics. I was definitely what one would call a "math cripple". . . I hated it, didn't understand it, and quit taking it as fast as my school would allow, which turned out to be after algebra I and plane geometry in high school. (Since then I have managed a course in college algebra, which was great fun and of tremendous value to me personally.) My initial interest and excitement with numbers came when I was handed a box of Cuisenaire rods the year I started teaching grade two back in 1960. The tremendous gains I made that year with the students, from playing with rods, really opened a whole new world for me. Since then I have been a participant in many interesting and profitable workshops in mathematics and have more recently been a staff member, a position that yields as much learning for me as I hope it provides for my "students"!

I have been fortunate to have at home four of my own junior Charbonneaus, who were young at the time I started with primary level mathematics, and with whom I could play games and "practice" lessons. But one's own children are not always ideal pupils with whom to work – for a variety of well-known reasons – so, while I strongly advise "trying things out with them," be careful of using them as "standards" for your students. My four are all so different, and react so differently to learning situations, that they have been a tremendous help, and I thank them for what they have given me through the years.

Attendance at workshops and classes in mathematics can be valuable and stimulating. Choose the workshop or course you wish to take very carefully. I tend to feel that short (one- to two-week) intensive workshops in mathematics which emphasize "doing" rather than listening are tremendously helpful. Pick your workshop leaders for their direct and active involvement with elementary-age children. This is not to say that a mathematics professor with a doctorate cannot be an effective teacher of teachers in an elementary math workshop! Several do work actively with young children as well as teach college-level mathematics, and are superb in their field. What I am cautioning against is taking a course in "new math" from a lecture approach if you are not strong in mathematics yourself. A class in which you are the active learner, working like a child, even at his level, in a discovery approach can be more beneficial. You learn to think like the child, even to anticipate the kinds of answers he will give in certain given situations to specific problems. If you can reach this point within yourself, you can then become much more alert to the kind of programming of problems, work cards, oral lessons, etc., you wish to use with the children. Working with a variety of people in the field of elementary math can help you see children in a totally different light, as well as to think about education in a totally different way from anything you have ever before experienced.

I guess what gave me the biggest fright during these years of learning (which I am still going through constantly with the children) was saying something or implying something or outright teaching something that wasn't true mathematically, and thereby leading the children astray for future math courses that they might take. A great help in this area is an upper school-level math teacher who is willing to give the time to watch me teach, and/or sit down with me and help me plan not only learning sessions, but worksheets and work cards. I have always found someone in my school most willing to do this. It really is a confidence builder!

The point I am trying to make is that if you are sincerely interested in learning mathematics with the children, there is always some talented mathematician around who is glad to help. Go to him for help, invite him into your classroom – there is so much to be gained on both sides!

Another tremendous help in learning, both for the children and myself, has been the tape recorder. By taping sessions with the children, I can analyze, perhaps with help from someone else, how I missed a "big idea," or asked the wrong question, or gained a new insight into a particular child or a particular idea with which we were working. If at all possible, work with a tape recorder going most of the time. The children soon learn to ignore it, and will continue their work as if it were not there. Try replaying a tape several weeks or months later and see what reaction you get from the children, too – often the point they were then "hung up" on has been completely cleared up, or they have taken a turn-about idea concerning the whole thing! It's quite remarkable what can be gained from "old" tapes.

I guess one of the key issues in working in a math lab with children of any age is one's attitude toward them and their learning. This seems like an obvious thing to say – it can be said of any classroom situation – but it is crucial in a math lab. It has taken me eleven years to reach this "feeling" about children and their learning capabilities. Despite what many experts in the field of education say, I firmly believe that children do not learn in an organized way. By this, I mean, in a way organized by adults. What is logical and orderly to us can be chaos and total mystery to a child. Each child is so very different, and each one must sort out for himself the logical approach to learning for him. Our job is to be sensitive to this child's needs and be ready to guide, to ask the right question which will lead him up the next step or around the next corner. But to presume to feel that WE, as adults, KNOW exactly what should be learned and when it should be learned, is conceit beyond all understanding. I am not in any way trying to imply that learning is completely autonomous – I don't begin to know the intricacies of a child's mind and thought processes. But I do know from these eleven years of observation and trial and error and success, that when left alone to learn in their own way, children do learn the necessary arithmetic skills and mathematical concepts. And rarely have two children learned them at the same stage of development or at the same rate of speed.

Traditionally, most children are taught operations with fractions at grades four and five. Many come through it with a certain mechanical proficiency; most forget it by the next grade and have to learn it all over, and too many come through dazed and shaken and hating math forever and ever! Why do we presume to insist that children MUST learn fractions at this grade-level? Maybe some can and should – Great! Let them. But those who shy away quite possibly need more experience, more background, more build-up, more working with "things" that allow them to see, feel, touch, work with – wallow in even – ratios, parts, halving, doubling, etc. I am constantly amazed at the youngsters who come up through the grades perfectly able to divide a fairly large number by 3, or take 1/3 of that same number, by a rote algorithm, but see absolutely no relationship between the two! Just a little careful programming of problems with or without "things" might have established neatly and clearly that dividing by 3 is the same as multiplying by 1/3. Which, by the way, is a far more useful concept to have securely in hand than the algorithm for adding fractions! Or something even more basic and simple – how many of us, or printed text books, teach addition first in grade one, then subtraction, barely touching multiplication or division, if at all? Why should a child necessarily learn addition first? Perhaps for some children it is an easier concept, but not for all. Why not learn subtraction first, or even multiplication or division? And here I am talking about the concepts behind these operations, not the symbols nor the writing of such operations – by working with Cuisenaire rods, for instance, all four operations can be learned simultaneously, with little or no confusion at all. Numerous other "playthings" can be used as well to look at these concepts in all possible ways.

Children do have an insatiable curiosity to learn, to "do" things, to find out for themselves, and with the proper guidance, they can be allowed to satisfy their curiosity, with tremendous interest on their part. What they are ready to talk about, they are ready to learn – it took me about four years to realize fully that children are about three weeks behind, on paper, what we have been working together on as a small group. They seem to need to feel their way, playing the games quite intuitively, coming up with things that work for some time before they are ready to tackle the same kinds of things by themselves with paper and pencil. Which rather led me around to trying a new approach with some. Why not throw out an idea, quite ambiguously, and see what happens; see what each child does with it, then be ready to step in with a suggestion, or get a small group together to explore some idea, or back off and try a totally different tack with a child who is completely at a loss with such open-endedness? And some children are! Some are terribly afraid of being turned loose to work something out for themselves. Most of them are just conditioned to "right answers," to pleasing some adult with no thought about what they are doing or "could there be another way?" or "what will happen if?". . . . Some children need more direction when first working in a lab just because they are conditioned to wait to be told precisely what to do and how to do it. Some have trouble distinguishing between freedom to choose a project to work on, and license to do as they please. Sometimes the answer to these children lies in work done in pairs or groups, so that they are drawn in and do become involved and even absorbed in what they are doing. Working with someone who is full of ideas will often stimulate the less adventuresome child into trying things too. When they are working on projects of their own choosing, and when they are confronted with a difficult piece of arithmetic, the work often seems less difficult . . . or at least it's more fun to solve, than it is when they are handed a page to do or are assigned some section in a book.

All of this sounds like complete freedom on the part of the child to do exactly as he or she pleases. Quite to the contrary – what any child does in a lab situation can be neatly controlled by the teacher, while the child is under the impression that it is totally his idea. The very bright, creative, capable child is no worry whatsoever; he will think up things to do, ideas to explore, difficult problems to solve completely on his own, stretching his mathematical ability daily. It is the timid, apathetic, or slower child who takes a bit more care and attention here, as in a traditional classroom. But the difference lies in that with patience and determination and conviction on the part of the teacher, this type of child often benefits even more than the talented one. How so? Because first and foremost, the pressure to "produce" – to achieve on a false level – is taken off. And I say false level because for so many of these children more time, a pace at which this particular child can operate effectively, not having to try to keep up with the class or even meet a curriculum "deadline" (we must be finished with our workbooks by ——), will develop a confidence and an ability to cope with the necessary arithmetic skills that all eventually must acquire to be able to live in this world and operate effectively. (And we must constantly keep in mind that the type of skills these children are going to be needing is ever-changing. We must see to it that the children get a twentieth century education!) These slower children must be given the chance to absorb like the proverbial sponge. What some do in days – these may take weeks to do, but it then sticks and doesn't have to be relearned.

Two statements from children make me see red: "I can't do that kind of problem because we haven't been shown how to do that yet." and "We learned that last year, but I forgot." If the child had ever really known, he would not have forgotten! And what seems to happen if children are started with a lab situation in grade one is that they develop into children who are not afraid to tackle any problem-solving situation on their own – whether or not they know the exact technique is not important. The attitude of these children seems to be: "Maybe I can't get the exact answer, but I can surely get close" – and

they proceed, by many ingenious means, to "get close" – remarkably so! To me, this child is the real mathematician in the bunch, far more so than the one who can compute with 100 per cent accuracy every time, but looks at you in utter blankness when you ask, "How many little white rods does it take to fill such and such a container?" or "How many ways can you cut a nine-square 'cake' on a geoboard into four equal parts?" Computational skills can be sharpened with all kinds of fun games and puzzles and races and contests, but trying to instill some sense of creativeness, adventuresomeness, curiosity, and confidence into a nine-, ten-, or eleven-year-old who, during all of his school career, has been quite content to sit and wait for an assignment, however well he does it, is most disheartening. This sounds like a wholesale condemnation of the child – not so, for most of them have been conditioned into this kind of behaviour by adults, parents, and teachers.

This is not to say, however, that upper elementary-age children who have never been exposed to a lab situation are beyond hope, help, and salvation! Again, with patience, and a firm conviction that this approach is a good one, these nine-, ten-, and eleven-year-olds can gain tremendously. How do I know? Two ways: from my own evaluation (and like all teachers, I feel I know my students best) – and from standardized testing. Yes, we do it; to prove to ourselves and the outside world that "way-out" ways of learning do produce the results that our society demands, and yes, our children come out exceedingly well. Most of the boys gain from two to five years in concepts and applications, and from one and a half to three or four in computation. (Stanford Achievement Test given in March each year.) What I personally look for on these tests is grade level or above in computational skills (great if they can score something like 8.3 in grade five, but really not essential at all), and at least two years ahead of their computational skills in concepts (many score four and five years ahead of their computational scores), and two years ahead of the computational scores in applications. In other words, I feel that I have been effective with them if their concepts and applications are well above grade level, or at least two years ahead of their skills level, and their skills level is what is expected for the grade. Of course, I get some who don't meet grade level skills each year. Even this is nothing to get into a real twit about; many, many times, this is a question of maturity and just plain "living," experiencing situations that require the use of a particular skill. Solving the problems that come up in very real situations rather than artificially contrived ones often serves as the greatest motivational feature one can hope for. This is in no way saying that the situations cannot be controlled so that the desired concepts are the ones being worked with; but the situation can be made to seem so real to the student (in many cases, a little cooperation from the rest of the staff in timing, etc., can make the situation very real, indeed!) that it is fun to solve, rather than just another task assigned by some adult which must be finished, like it or not. Again, mathematics does not have to be practical in the sense of "grocery store arithmetic" that is the dullest of the dull, but children DO enjoy solving the problems that arise in many daily situations – and especially enjoy having a part in creating these problems. Some of the work cards reproduced in this book will give you ideas about practical problems for creative solving. (See Appendix.) And the more experience children have with creating their own work, the more work they will create, and the greater progress they will make. I have seen it happen, time after time, to the point that I now have the utmost faith in children and their learning.

Like all adults who have grown up under a traditional system, I have found myself feeling guilty in classroom situations where, perhaps, the children have not done steady practice in the four operations in increasing degrees of difficulty. I have even felt panic-stricken to realize that they must face standardized testing, and "what if they don't know how to do . . . ." and I have dutifully assigned practice work, with "so many problems or pages" to be turned in to me daily, with an admonishment to the children of "you must do . . . ." What has resulted each time is a situation of teacher fighting student, force against force, and let's see who wins. Yes, I can make them do worksheets full of problems, I can keep them in from recess, or after school, or for extra help sessions or study halls, and what results is that I feed my own ego – I have accomplished the work set out, I have made them do the practice I felt necessary, I'm a tough person who demands a lot of work! Yes, I have TAUGHT, but there has not been much LEARNING going on. Time after time, I have seen the same children who fight hardest against deadlines and assignments do twice as much work on their own when told, "Go as far as you like or can; do as many papers as you wish or have time for – come and ask for help if you need to, or ask your buddy for help. Maybe he can help you see where you goofed better than I can." Children like to have some control over their lives instead of constantly being told what, when, why, where, and above all HOW. The little one who constantly lags or does little has very real reasons for behaving in this way – and maybe easing up on quantity, on the degree of difficulty, can make the difference between tuning him out completely and giving him just the kind of help he needed at the right time. Backing off and staying out of the way can be as much help as a personal tutoring session at times – maybe more!

I know what is going through many minds at this time as you read this: What happens when they go on to the next grade, and particularly what happens when they go to junior high school and the grade seven teacher asks "What happened in sixth grade – didn't that teacher teach anything? Why is this child passed along to seventh grade when he can't do . . . ?" Yes, I've heard it all before, too, but the question that

must be answered is do we as teachers do what is best for each child, or do we do what is most expedient for us (teach the class as a group and hope for the best) to meet the next grade-level demands, whether the child is ready or not? How many times have we all heard the statement about children not developing at the same rate, mentally or physically? We all believe it, philosophically, but the minute they get to school we classify and pigeonhole and lock them into step and pass them along, restraining some and dragging along others. WHY? Because it's the easiest way to do it. We even talk of efficiency – teaching or having all the children in one class working on the same thing at the same time. Isn't it more efficient to do it this way? Efficient for whom? For the teacher, yes, but really and truly, how many children are REALLY ready for the same concept at the same time? If they were, would there have to be so much reteaching and going over and "review"? I shall not try to speak for older children, but certainly elementary-age children desperately need to develop at their own pace – some can and should learn long division at grade four, but others can't cope with larger number concepts yet, and waiting until the ripe moment can save hours and days and weeks of agony for young minds and frustrations for teachers. Most teachers do know their pupils very well indeed, sometimes better than parents, even, and it isn't difficult to learn to sense when, just precisely when, this child is ready for, say, three-digit multiplication and two- or three-digit long division, or fractions. As far as passing them along to the next teacher is concerned, it certainly isn't THAT hard to sit down and talk about the children who move on, their strengths and weaknesses, their interests and abilities. It isn't hard to pick up each child exactly where he is and take him as far as he can go each year. Some years will show tremendous growth, some years not so much – just as in the classroom, some days or weeks, great leaps are made, others seem to show only a snail's pace in growth. We, as adults, operate very unevenly in our daily jobs, be it teaching or whatever. Some days are just great, we have tremendous energy and everything seems to flow – others just don't happen like that. Can't children have the same feelings and frustrations? Some days they want to work and do things to peak capacity; other days may need to be thinking days, or exploring days – or even daydreaming days.

I think one of the most difficult things for me has been to learn when and how to correct mistakes. One thing I KNOW – it is not with a red marking pen and large red X's all over the paper. The most effective method for me has been to keep constantly moving around the room while the children are working, and if I catch a mistake in the making, I can say something to the effect that "I don't think I agree with that. Can you show me how you reached that figure?" or "You work that one over again and I will, too – and we'll compare what we get. Yours is different from mine, and I may have goofed somewhere." We talk a lot about sensible answers: Does that make sense? Is it logical, considering what you know? I learned the most about when and what to correct from a little guy I had in Kindergarten mathematics, then again in grades one and two. He was a very little boy, physically, and quite timid. Mistakes marked on his papers in grade one brought nothing but tears and total fear. Nothing was learned by this little boy from correcting mistakes on such a paper – he was just dissolved and withdrew completely. I found that by carefully working around near him, I could watch what he did with his Cuisenaire rods to reach the "answers" he got, and by saying such things as "Show me how you built that," or "Let's build that one again," and talking lots about numbers, and playing games with rods, but never, never marking anything on this child's paper, I could help him to gain confidence in himself and me. He began to make fewer and fewer errors, and by the time he had finished round one in grade one, he actually could smile and have some fun in math. I say "round one" because this lad did repeat grade one. Because of his size, age, and immaturity we felt that he could greatly benefit from not moving ahead at this point. He had just begun to gain some small bit of confidence, and the challenge of grade two would demand too much stretching on his part at this point. Round two in grade one put our little friend in the top sections of reading and math. He was just that much ahead of his peers that he really began to feel that exhilaration of I CAN REALLY READ AND DO MATH, TOO! I began to find some few days when I could actually come along and make an arrow with pencil next to a problem he had done incorrectly and say "Don't think I agree, can you prove it to me?" And he would rebuild the problem with rods and find his own mistake. Basically, he could do all his worksheets (and we were using a lot of our own plus Lore Rasmussen's Math Lab sheets) [2] with about 80 per cent accuracy. I started using a method of putting a check on top of each paper, which said to the children: "I've looked this over very carefully, and you seem to know what's going on – good work." If the child missed the point completely, I could have another worksheet with the same concepts on it ready for the next day and work on it with him, rather than give back a paper full of red marks and say "Correct your mistakes."

Grade two for this lad turned into a real triumph. He became very adventuresome with equipment, designing his own number line with a yard stick, and finding many interesting ways to do all kinds of mathematics. We even reached a point during the last part of the year where I could say "You goofed – go try it again!" and with a grin a mile wide, he would, and find his mistakes!

Working with this child for four years taught me so very much about children and how they think and particularly how they react to adults and correction. I shudder to think what I might

[2] Mrs. Lore Rasmussen, 8012 Winston Rd., Philadelphia, Pa. 19118.

have done to this child if I had persisted in using my previous more standard way of marking papers. By the end of grade two, he could "take" correction and could even tolerate a red arrow on his paper – an arrow to indicate I didn't agree. I say "didn't agree" rather than "wrong" because oftentimes I would find that he wasn't even doing the problem that was there, or at least hadn't interpreted it in the same way I did. Often the answer he had arrived at was correct for the problem he had done. By talking it out, we could come to a meeting of the minds. This child was bright, creative, and thoughtful, when given the time and the respect of the adult to work out situations in his own way. Yes, he went on to another math teacher in grade three, and different ones from that year on, with major success all around. He taught me so much, and I am very grateful, for I began to look at all my children as individuals, real individuals, with strange and different learning patterns. It was my job to find these patterns and deal with them successfully. I have developed this system of checks at the top of the page, which I use almost exclusively now – the children all know and understand what it means; and at any time, if they wish to see an exact score, I give it to them. However, scores mean relatively little. No child should be getting constant 100's and perfect papers. Life isn't like that, it isn't realistic to go through school with perfect papers. What a shock to get out in the world and find you're not perfect! Children should make mistakes; they MUST make mistakes. This is the greatest learning situation that can be created, but we must be supersensitive to HOW we correct them, or rather get the children to correct them.

Two quotes immediately come to mind:

> When a child is denied the privilege of making mistakes, he is at the same time denied access to one of the best learning experiences. (Editorial, Volta Review)

and

> School should be a place where pupils can make mistakes without excessive risk. Where they can experiment, try things out, test themselves, test others, test ideas and feelings and impulses. (Don Robinson, Scraps from a Teacher's Notebook)

I wonder about this gentleman's expression: "School should be . . . . " How many schools can honestly say this of themselves? And how many teachers? It seems to me that we are still geared to meeting arbitrary standards with all children at all levels, and while we very much believe, or say we believe, in individualization of education, not much is being done to achieve these goals. Children still meet an age standard for entering school, we get "up tight" when little first-graders fail to learn to read (and progressively more so as they become second-graders and third-graders and so on, and the problem still exists). No one says, perhaps that child should have never started to learn to read at that stage; maybe, he shouldn't have started school at all at that age.

It really cannot be that difficult to work out a plan for allowing children to start school at different times and learn at different rates. Take a careful look around. Is our traditional system of teaching really producing children who love school, who love to learn for learning's sake, who are independent and eager enough when they arrive at school in the morning to get right busy with a project, or something in which they are absorbed, and work at it with vigor, for an extended period of time? Are we producing children who speak well and have interesting things to say, who write fluently and with obvious pleasure? How many of us have observed a classroom where the children are all busy with such delight in what they are doing that you can feel the electricity of excitement ripple from corner to corner – and where children are bubbling with ideas, and one solved experiment leads to another (perhaps not in the logical order WE would have chosen), and where the teacher is so much in the background that you are not sure there is even an adult around? Yes, this does happen, and with small children, too. I have watched it, and experienced it, and I know it can be successful. Really and truly, to borrow a phrase from the master, Caleb Gattegno, it is "the subordination of teaching to learning." I can remember hearing of such exciting classrooms quite a number of years ago – in England – and I guess I believed it, but strongly felt that it could not happen here in the United States. Our youth did not have the respect for authority, for adults, that British children had. But I was so very wrong. In the summer of 1966, I had the great experience of working in a workshop in Cleveland, Ohio, with ghetto children and the teachers who faced them daily in their winter sessions. A team from Leicestershire, using the methods of the Integrated Day, was the core of the workshop for six all-too-short weeks. I saw children who were tuned out and turned off by a traditional set-up suddenly become alive and interested and make such gains in all disciplines that it was fantastic!

In the back of my mind, gnawing away, was the dreadfully uncomfortable feeling in my own school that I was witnessing a great deal of brilliant teaching, but not much learning. I mean "brilliant teaching" quite sincerely; beautifully structured courses with interesting people teaching them, good books and teacher-created materials, well thought-out lessons that progressed neatly and logically from one concept to another, building the necessary background for the next level of difficulty. And I saw children politely paying attention, sometimes looking as though they were even quite absorbed by what the teacher was saying, or answering the well-structured questions correctly, giving the impression that the class was really making the discovery intended for that session. But let the

teacher turn them loose on worksheets which involved that just-discussed concept, and quite obviously many had missed the point. When the bell rang dismissing classes, they roared from the room like prisoners with the cell-block doors just opened. All of this made me feel quite strongly that what we were doing was not right. – Yes, we were teaching, but in all truth, there was not much learning. And certainly none of us could claim pupils who were so fascinated with our subject that they weren't ready to leave at the end of forty-five minutes. That was my state of mind when I was first exposed to the Integrated Day as an environment for learning. I saw it happen. I saw it make of children fascinating little people whose thoughts and ideas suddenly became important and interesting to them and to the adults who worked with them.

I have, during this summer of 1970, just finished four weeks as a staff member of another Integrated Day workshop at the Academy of the Sacred Heart, in Bloomfield Hills, Michigan, sponsored jointly by NAIS and the Academy, and I am more firmly than ever convinced that this is the real environment for learning. And I am forever grateful to John and Janet Lancaster, our directors from England, to Jock Denio, Michael Butler, and Carolyn Tower, fellow staff members, and to Sister Anne Hills, our administrator, for reaffirming my good feelings about this approach. As I have said before, all of this sounds like complete autonomy on the part of the child to do as he pleases, learning what he pleases and when he pleases, taking no direction or lead from a teacher in any way, with the result that everyone ends up going in a different direction at a different rate of speed! (I'm not so sure this would be the chaos it sounds, but this is not what I am getting at, at all.) Looking just at a mathematical laboratory as a learning center from the beginning: there are, of course, certain concepts and skills in arithmetic and a lot of mathematics that children need to know, but they do not need to learn them together, at a time which WE prescribe is right for "all ten-year-olds." Really, what is wrong with learning division first before addition, or subtraction, or multiplication?

Within the given space of time that children go through the elementary years or grades, there is certainly time enough to get in all we feel they should acquire. All I am arguing for is the right of the child to clue us in as to the ideal time for HIM to learn a particular skill or concept. And they do clue you in; if you watch carefully and listen even more carefully, you can sense when this one is ready for a long division algorithm and that one is ready to start learning operations with fractions, or begin to feel his way through decimal notation. Maybe one can do it at eight years old, another not until he is ten or eleven. What is the tragedy of working at a level one understands and can cope with, rather than being forced to go through the motions with an idea that is so foreign to one's experiences that he cannot possibly perceive the structure, or understand why something works? It is when children are pushed beyond their limits that they become angry and difficult, sometimes sullen, and sometimes so far withdrawn from the situation it's as though they are in one world and teacher in another. We as teachers don't listen to the children at all – we may think so, but not really.

An incident I observed in a third grade classroom keeps running through my head: a group of little girls were doing short division problems on the board, and when a mistake was made, the teacher kept repeating like a broken record: "Girls, we don't know our facts!" After quite a few minutes of this, at one point she burst out, "Now, girls, remember that when we multiply, we always get a bigger answer, and when we divide, we always get a smaller answer." One little thing sat there, shaking her head, and finally she raised her hand and said: "Oh no you don't, Miss ———. When you divide, you have the same number of things, just broken up into different kinds of groups."

This child's concept of division was quite sophisticated and certainly was worthy of discussion on the part of the bunch being worked with, but teacher's answer was given with a benevolent smile; "No, dear, when you divide, you always get a smaller answer." She no more heard what that child had said than she could fly, and furthermore, she hadn't even heard herself, for after class I could not resist asking her how she expected the girls to feel when they worked with fractions and discovered that 1/2 x 1/4 equaled 1/8, which was certainly smaller than either 1/2 or 1/4. And how about the situation of 4 divided by 1/2? Her answer? "But, I don't teach that level of arithmetic."

It is just little things like that that can cause so much trouble with children and mathematics. It's no wonder when first confronted with 2 ÷ 1/2 most children will say 1. I've had little six-year-olds able to work out that problem just because they understand such division problems as asking how many 1/2's there are in 2. I am not in any way saying that all six-year-olds can and should be exposed to a situation like that, but those who can handle it should be working with it. And I guess what I am really trying to emphasize is being so careful of what you say to a child. Don't tell him something that will have to be undone later on – and don't let him have the impression that you are all-knowing.

One of the classic examples of misinformation that many youngsters come through the elementary grades with is in subtraction which requires "borrowing" or "regrouping." The problem might look like this: 724 - 569 = △ and teacher proceeds to say: "Now you can't take 9 from 4, so what do we do?" Well, you can too take 9 from 4, and most children realize it, too. If regrouping is the topic, certainly some other way of expressing the idea can be used than to say "You can't take 9 from 4"! I have watched a seven-year-old do that type of

problem beautifully like this:

$$\begin{array}{r} 724 \\ -569 \\ \hline 200 \\ -40 \\ \hline 160 \\ -5 \\ \hline 155 \end{array}$$

Beat that for original thinking! And if you don't understand some rather unusual bit of reasoning on the part of the children, don't be afraid to admit that you don't get it. My children show me lots of things that stump me, or ask me things I can't answer. If I think I can help them find out, we work on it together, find books where we can maybe find out, or I send them to someone who can help – an upper school math teacher, for instance! I make sure that the children know I don't know all the answers, but we can sure as heck find out somewhere.

I can remember a junior high math teacher saying at lunch one day, in complete horror: "I made a mistake today on the board. What am I ever going to do now? Those kids will never respect me again!" The trouble is, with that attitude, they probably never did in the first place. Children don't lose respect for an adult who makes a mistake; honesty is a key word in working with the young. They can sense a phony in minutes, and the game is on to see how much they can annoy, where they can irritate the most, how far they can push – and that's what school is to many children: a large-sized game. See whom you can annoy, and how much work you can get out of. Frustrating? YES – to both students and teacher.

I don't claim that a math lab will miraculously cure all ills, all discipline problems and all failing students – particularly when started at the upper levels with no such experience in the primary years, but it is NEVER too late to start. It will just take a little longer to jell and really move, just as starting at the junior high level makes it that much more difficult when the children have never faced it at the elementary level.

If starting at about the grade four level for the first time, work slowly. Start a "lab session" perhaps once a week for a half an hour – maybe pick that last half hour in the day when everything drags. The children need to relax and so does the teacher. Start with some interesting mathematical games like QUINTO and HEX and chip trading, or VECTOR. Maybe cross number puzzles, or the TOWER OF HANOI or the peg puzzle or KALAH – all things that get children thinking from a logic and strategy standpoint, but which don't look like the usual "school" stuff. As they begin to find things to do without suggestions from you, stretch the time a little and start another session on another day. Gradually build up the time you allot to math lab activities to longer and longer periods, and more of them each week. Start collecting ideas that come to you and the children on 5" by 8" file cards – things that are fun to do mathematically, and soon you will have a ready reference file all your own. Take particular care to record the things that come from the children; as they work on one thing, it often leads to another. Add materials as ideas pop up: games can be acquired through birthdays that come during the school year. Instead of that "party" with cookies or cupcakes, etc., encourage the children to give a game to the math lab in honor of their birthday, and paste a sticker on the box with their name and birth date. Don't hesitate to let children just read, or even just sit and think – all of this can lead to something quite interesting and profitable.

The boys in my lab have asked to work on social studies projects, reading, researching, making maps both two- and three-dimensional, scale models (what can be more mathematical?) – which has in some small way made the children realize that education is learning about all kinds of things, and that in so very many ways these things correlate. School is not just a bunch of subjects to be thought about for three quarters of an hour and then turned off as the next "subject" comes up. It is learning in all its phases and very much integrated. Art and math correlate beautifully, as do all the subject areas, really. Writing up the results of an experiment or a work-card in math or science becomes a way of communication with others about what you are doing, not just an exercise in English to see if you know a noun from a verb. And trying to find out more about Pythagoras' theorem, and other things he did, often leads to reading extensively in the biographies of other thinkers of the age. As you become more comfortable in this role of observer and guide instead of leader and "teller," gradually expand the time, the topics, and the materials available for use, until both you and the children feel quite at ease in this kind of situation. Be aware that, of course, things are going to be considerably noisier than in a traditional classroom, that children should work together in twos and threes and sometimes larger groups, and that their talking is not wrong. Listen carefully to the chit-chat – many times I have found a child much better able to explain something, or to express a thought, than I; or have even heard a peer group argument, not refereed by me, become a means for a real discovery into the meaning of a concept. How much I wish I had had the tape recorder back a few years ago in grade five, when I had turned the group loose on projects, and two young lads had chosen to work with volume (without knowing that this is what you call it)!

They had become quite intrigued with the idea of how much various boxes and cartons held, and had progressed from quantities of sand or liquid to using centimeter cubes and inch cubes, depending on which fit best. They were working with a cardboard gallon milk carton, and had started filling it neatly with centimeter cubes reaching about three layers deep. All of a sudden, one of them said: "Gee, this is a stupid way to do this! I can think of a quick way to do it with a centimeter ruler – all we have to do is measure the sides and multiply." The other one agreed; they got the centimeter ruler and pro-

ceeded to measure the length and the width. From then on, there was no agreement at all. The first little guy, whose idea it was, wanted to measure the height, width, and length, and multiply the three figures together. Boy number two insisted that all the measurement needed was the width of the bottom times the height of the side, times four and add the square on the bottom. A real argument ensued! Boy number one could not convince boy number two that he was wrong, considering what the problem was (which was to see how many centimeter cubes fit into the carton). It was the classic argument about volume versus surface area, and neither of these boys, being exceedingly bright, would give in. Their decision was each to do the problems with about six different cartons by filling carefully, and then by measuring, each in his own way, and see what happened. Boy number one was finished in about ten minutes and was quite sure he was correct, because both by filling and by measuring, he arrived at the same answers. My other little friend also filled each carton carefully, then measured "his way" – and strangely, the answers didn't match! He knew the number of cubes found by filling must be correct, and when he checked with his buddy, found out why the first boy's ideas of measuring were more valid than his. By actually doing it, he became convinced that his friend was right, and that his own view of the problem was slightly awry. However, the two of them began to wonder if there was any relationship between volume and surface area and went on to do quite a bit of experimentation along these lines. Their conclusions were that they could find a pattern to surface area and volume with cubes, but it was beyond their scope of understanding with rectangular solids at that point – but they felt there was one!

How does one keep records of what is going on and what is being learned by each child? I have found that the easiest way for me is to keep a three-ring notebook on the children I have for a particular year, with a sheet of paper for each child. I can easily record immediately after lab any exciting piece of work the child has done, or any blatant errors that are being made, so that I have a fairly complete idea at the end of any given month about what "Johnny" has done or worked with. Perhaps nothing other than normal steady progress is being made; then, perhaps I won't have an entry for a particular child for several weeks. This is the clue to me to make some rather "way-out suggestion" for experimenting, to get him going outside of more routine tasks, and perhaps this period of slow contemplation is just what was needed at this point in the child's development. Anyhow, I record what impresses me, good and bad, and I keep accurate records of quiz scores and computational skills levels. Yes, I "quiz" in quite a traditional manner, with quiz sheets involving only computational skills appropriate to what most of the children can handle at that stage. It is an accurate clue to their development, and is useful for me to know who needs help or "programming" in a specific area, maybe some more background and readiness-building before tackling that kind of problem again. And I do this fairly often, weekly or every ten days or so, thereby keeping in close touch with the skills area. This can also be done by just watching the children solve the skills problems that come up as they work on their own projects, but one does need some basis besides this, I think. I keep quizzes filed away in a manila folder for each child, so that I can refer back to the type of mistake he has made and try to keep it from happening again. Just having the scores is of very little value; you need to know "what" the error was and try to see what kind of thinking led to that conclusion, so that you can better help the child to a more logical approach.

Don't underestimate the power of the camera as a recording device. I have used a Polaroid colorpack to great advantage, as you can immediately see if you caught what you wanted; but slides are also of great benefit. I take pictures of children in action, the work they are doing, and the finished product, whether it be a sheet of paper recording "what happened when," or a scale model bridge, or something they have done in full color. Sometimes I can get the children to donate the finished product to the math lab, and each year I try to have them make up a scrap book of things they did – which is great fun for me, as I have some fascinating "proof" of the exciting things they can produce, and a pretty accurate record of the year. And the children love to do it.

Another successful way to record individual progress is on 5" by 8" file cards, which can be kept easily on your desk and added to daily or weekly, as you wish.

Let's think about mistakes again for a brief time. Some of the most beautiful lessons can come from mistakes, both your own and the children's. I can remember a little guy some years ago in one of my second grades who resisted the discovery method completely, and no matter what we worked with together on the board at school and discussed and "found out," he went home and asked his father "how to do it." Daddy obliged beautifully, and showed him method after method, which junior brought gleefully to school and almost daily proceeded to try to show us "how to"! The only trouble was that junior had grasped only about half the steps, and consequently couldn't come up with much beyond total muddle and chaos. From his confusion, we built some beautiful lessons; his classmates were able to see through what he was being told to do, and adjust the techniques to match the idea. Our little friend wasn't able to make this kind of sophisticated step yet, and became rather hopelessly lost. And my guilt lay in that I saw only what the rest could gain from using his mistakes. Strike one child off to a very bad start in mathematics because of my insensitivity to his needs at that time! I have since learned, I hope, to take a mistake and turn it into a profitable lesson for others only if it does not put the person responsible into a position of ridicule.

An example of this kind of helpful chaos – using a mistake to good advantage – occurred just shortly after we had worked up some ideas for developing the multiplication concept and had sent our handwritten papers to the office for dittoing. One of the directions read: "Take a handful of rods. See if you can make a square. If you can, write down how many rods you have in each row, and how many rows you have." The children were having a terrible time trying to do what was asked. Frustration reigned, and tempers flared. I kept urging them to keep trying, and finally one of them actually did get a square. When we looked at it, and figured out its dimensions, it turned out to be a square of eleven by eleven centimeters, made up of all different colors in each row, so all we could do to record it was to say we had eleven rows of centimeter blocks, with the equivalent of eleven centimeters in each row. I asked the children if they wanted to throw away the worksheet and try something else. Their reaction was "NO – let's rewrite it to make sense and try it again." One suggestion was to insert the word "white" on the first direction, so that it would read "Take a handful of WHITE rods." This would make it abundantly clearer, and in fact, we found it was what should have been there in the first place, when we checked back later. But some of the children decided the problem would be much more interesting if it asked the students to see what kinds of squares they could build using each color of Cuisenaire rod in turn, starting with whites.

> What is the smallest square you can build with white rods?
>
> Will one white rod equal a square?
>
> Will two white rods equal a square?
>
> Will three white rods equal a square? etc.

And the recording was to be done like this:

1 white makes a square or 1w = □
2 whites do not make a square 2w ≠ □
with □ meaning a square

with each one's being built until a pattern began to emerge and the child could predict what would make a square and what would not, and then proceed to use red rods and the same technique of building and testing again until a pattern emerged, and so on with each color in the box. By the time the children had reached the dark green and black blocks, many could predict exactly how many would work, and would write down only those number of rods that would make a square. Then they began thinking in terms of white rods only: for instance, if six dark green rods would make a square, this was equal to 36 white rods, and began to see quickly that the squares in colors could all be done in whites; in fact, they had all of them on their first sheet. But by doing all the colors, they learned much about how squares grow, and about number patterns and sequences. AND LOOK AT ALL THE COMPUTATIONAL PRACTICE THAT WAS SNEAKED INTO THE INVESTIGATION! Most of all, a GOOD lesson was created out of utter chaos, and no one was hurt in the process.

I would like to discuss a bit further two words which have heretofore been taboo in any elementary classroom: chaos and frustration. Schools of education and master teachers alike have usually spent hours telling prospective teachers of younger children that calm, quiet, organization, control, and certainly no frustration must exist for proper learning to take place. Nothing could be further from the truth! Real learning comes as the result of frustration, trial and error, mistakes made with experiments, and further exploration to profit from those mistakes. Every child has a frustration tolerance or level – and this is what a teacher in an open classroom looks for – just how much error can the child tolerate before he gives up? And there are times when "giving up" at that point may be the best thing to do – keep the result of what happened so far, tuck it away for a time and perhaps all of a sudden get an idea, or learn from some other project being worked on, some little clue to further the original piece of work.

The world we live in is chaos and frustration. Children live in it every day of their lives. Frustration and chaos are the keys to allow them to sort out some kind of sense from the real world. We cannot and must not protect them from this kind of learning experience. Our job as teachers is to see that the child is not pushed too far too fast, nor given too little too slowly; this kind of frustration is damaging.

An open classroom is not quiet – that is, silent. But normal conversation and the sounds of work going on should not upset any child. They have the ability to blot out anything they do not wish to hear, and if a child must have real quiet for some piece of work he is doing, a quiet corner in another room or in the hall can be found. How often have you gone through a lesson with a whole class, step by step, with all of them looking very intent, and when you are through, found that not much could be repeated back to you? I have, many times in past years! Children tune out adults with the greatest of ease, or each other, for that matter. Noise, productive noise, does not bother or annoy. The deliberate disrupter, the silliness, can be stopped quite easily – no lab work, no choice of projects for that one!

I would like to quote from an article in the Saturday Review which appeared on May 16, 1970, "A little Bit of Chaos," by Beatrice and Ronald Gross:

> Piaget is best known for his finding that intelligence – adaptive thinking and action – develops in sequence and is related to age. However, the age at which children can understand concepts varies from child to child, depending on his native endowment and on the quality

of the physical and social environments in which he is reared.

But Piaget's books – The Origins of Intelligence in Children, The Psychology of Intelligence, and The Construction of Reality – based on his research on how children learn, also proved that it is a waste of time to tell a child things that a child cannot experience through his senses. The child must be able to try things out to see what happens, manipulate objects and symbols, pose questions and seek their answers, reconcile what he finds at one time with what he finds at another, and test his findings against perceptions of others his age. Activity essential to intellectual development includes social collaboration, group effort, and communication among children. Only after a good deal of experience is the child ready to move on to abstract conceptualizations. Piaget is critical of classrooms where the teacher is the dominant feature, where books and the teacher's talking are basic instructional media, and where large group instruction is the rule, and oral or written tests are used to validate the whole process. Clearly from his findings, traditional techniques are ineffectual. But for children who must depend on the school environment as the richest they are to encounter, it can be downright damaging; denied a chance to grow, their minds may actually atrophy.

Watch the children you teach after a few months of open exploration and choice. I think you will be amazed at the learning going on! I have had many traditional teachers react with "I can do it faster my way" or "How can you stand by and let them make mistakes?" or "Couldn't they learn faster if you told them?" The answer is emphatically NO! Some few might pick up whatever technique it is that you are demonstrating, but most don't get it at all, and you find yourself going over and over and over – ad nauseam, and the children just close their ears that much tighter. A child learns at his own rate – and your rate and method are not his. Help him, be ready with all kinds of ideas and suggestions to make that next step, but don't force it. Don't "TEACH" – let them "LEARN"

There are many, many things that can be worked with in a math lab, and there are a number of things that are ideal. Again, one of my criteria is the degree of independence that one can have with a particular topic; this often determines whether or not I will use it. With any topics, there are, of course, some children who cannot start out with as much lateral freedom to work as others, who just haven't the imagination at this stage in their development to begin on their own, nor the ability to take a subject very far in depth. For this reason, the work cards in the Appendix are dealt with in two ways: the first one in each set (yellow) is very open-ended and free, for the children who champ at the bit and can't wait to get going with something entirely "their own"; and then, the same idea is developed in a more structured way, with several cards (green) instead of just one, for the children (and teachers!) who need more help to get going, and possibly more guidance along the way. I have deliberately kept the directions exceedingly simple and the reading down to a minimum on each card. It has been my experience that children, no matter how bright, and no matter how creative will not read work cards loaded with small print and fine detail! If there's lots of reading involved – FORGET IT! Once they get under way, they do not seem to mind reading in detail; they will often find that their project takes some researching in the library, but not initially!

Aside from shying away from the printed word as such, I think what the children really want is contact with the teacher – some indication that you really care about what they are doing and just haven't shoved work cards into their hands to get them out of your hair for a while. Also, if the directions are not too explicit, more creativity in approach and recording comes about, and you will have to keep in touch with what is going on, and perhaps make a suggestion here and there to get the project moving in more depth. I cannot emphasize too strongly the need to keep in direct touch with what each child or group is doing, for many times, I have seen children go through very specific work cards, with beautifully programmed ideas for the development of a concept, and they have answered each specific question correctly, but at the end have missed the point completely. If it's a "big idea" you want to develop, make sure the children get the feel for where they are heading; they don't need to know the precise goal the minute they start out, but as they work, begin to ask questions that lead in the right direction. And be doubly sure to discuss each project as it is finished with the children involved. I do not often discuss a project with a whole class at once, and for several reasons: one is that they're not all interested, and even though they may look as though they're paying attention, they may be a million miles away. Therefore, you haven't reached them anyhow. Also, another group may wish to pursue the same work card on a later day, and if they haven't been "clued in," may do it in an entirely different, but equally successful, manner. Toward the end of a trimester, or semester, or even the year, it's great fun to get some discussions going comparing approaches and finished products.

Remember to keep moving about the room, asking for an explanation of what this or that means, what are you doing here, what do you think you will try next, perhaps making a suggestion instead of asking a question. And don't hesitate to work away quietly in a corner on a project of your own – particularly if it is one that fascinates you, and none of the children has as yet, attempted it! I can often get something going by just sitting down and starting off, and soon I have a small collection of little souls who

want to know what I'm doing, etc. And sometimes I will work through a whole project this way, with two or three helpers. My good friend from Leicestershire, Roy Illsley, said to us during the workshop this summer that it is good for the children to see us at work, going through the same frustrations with things that don't work, or skills that need perfecting (such as me and the slide rule!).

If the children are used to having you wander around, they don't mind a bit being watched, as long as it is in the spirit of "Gee, what you are doing is interesting! Mind if I watch a bit?" And don't be too upset with some of your students who watch a while, too. Sometimes they take in so much more than we give them credit for, and sometimes watching can be a valuable build-up for taking the next big step on one's own. A constant "watcher" can be enticed into participating by asking him to help get materials ready, or make something for the younger children that will help them with some needed learning situation. Some of my sixth-graders who tended to "watch," got very involved in making a "stretching machine" when I asked them if they could help me think of some good game to work with the seven- and eight-year-olds on multiplication. They had been doing the UICSM program, Stretchers and Shrinkers, in grade six, and had had so much fun reviewing multiplication themselves with a stretching machine on paper, that they decided a model stretcher would be great for the little ones, one into which they could actually feed Cuisenaire blocks, and if they asked for the right "output," would get it out the other end! Of course, this "machine" required the use of one of the eleven-year-olds to handle the input and the output, but made an excellent "thing" for the little ones and helped the older ones as well. Since the original stretcher, has come a shrinker – obviously for division – and a rod chopper and gluer for addition and subtraction.

Some of the most ideal topics for a mathematics laboratory are measuring, weighing, concepts of fractions, velocity, simple machines, other names for numbers, map making and scale drawing, as well as 3-D interpretations of the drawings. These sound very simple – and they are – but there are so many variations that can be worked within just those few topics! I shall go into quite a bit of detail on each of these subjects in the work card section. With measuring alone, you can get the children to work with linear measure, surface area, volume with many, many nonstandard units of the child's own imagination, and then with standard units of all kinds and varieties. Also, estimating heights by the shadow method and using a 45-set square. Simple topography can be done with a telescope and a stadia rod.

Weighing again can be done with nonstandard units to begin with on a simple pan balance, proceeding then to gram and ounce weights with more delicately balanced scales. Estimation with both weighing and measuring is an extremely important concept to build up, and can be done easily with children as they love to "guess" and see how close they come!

Actually, there is such a wealth of material in measurement (with weighing classified as part of measurement) that a whole year could easily be spent on this alone! I once heard a speech at a national convention on "Measurement – a Laboratory for Learning" which was excellent in every respect and certainly very, very true.

Fractions can be explored with drawing, cutting, folding, building, taking things apart. And velocity can become real with the use of electric trains, battery- or gas-powered trucks and cars. Simple machines provide excellent material for graphs of data which enable children to predict and find "rules" that always work – even coming close to the real thing at this point is thrilling and rewarding to the children, and reinforces the correlation between math and science beautifully.

Map making and scale drawing give real meaning to social studies and stories read in English – many projects can be correlated in these areas. One of our groups of eleven-year-olds had read Boy of the Islands, and proceeded to do the most beautiful and detailed drawings of the islands, with reefs and scales of miles, and perfected a large three-dimensional representation of their work. In many ways, they actually "lived" the story.

Many things come to mind as I list "ideal topics," but what I wish to provide here are some ideas with which one can get started; not provide a full year's curriculum in math lab topics! This would be very difficult to do for someone else – to repeat myself again, no two children are alike, and no two school situations are alike. What might be great for my boys would not be applicable at all in other classes. Sequencing topics would be almost impossible; the children will do that for you. "WHAT THEY ARE READY TO TALK ABOUT, THEY ARE READY TO LEARN!" Listen carefully; they will let you know what they are ready for, and when they are ready they will eat it up! In the meantime, build in the readiness, keep them intrigued and interested, lead with the "light touch," and they will reward you with the most satisfying teaching experience you have ever had.

A word of caution about the cards in the Appendix. They are not intended to be used exactly as presented. Nor are they intended to be used, each and every card! Some ways of covering a topic may be more appealing to you than others. Don't hesitate to scrap any cards you don't like. Neither topic nor sequence is that important. Use your own best judgment for your children. Number line work need not precede "other names for numbers," or measuring need not come after work on geoboards. I stuck them in the sequence you find here, for variety and interest and change of pace. The topics are definitely to be used as you see fit with your

class. Each topic, however, is divided in the following manner: The yellow card in each set is for the GO GO GO kids, and the green ones are programmed into easier and, I hope, more logical steps for the less secure children. In most cases, there are two or three or more green cards for each yellow card. The yellow cards can be used with all the children after they have caught the idea of a freer situation and have begun to blossom – use them as a sort of "No restrictions now, kids, GO!" type of approach.

Don't hesitate to rewrite or reword or in any way improve the work cards for your bunch of children. These are to be only suggestions, some thoughts to get you and the children started in this open situation.

A word about the format of the cards: you will notice that they are handwritten. My boys seem to prefer this; a bit more of the personal touch than the printed word, I guess. I have tried both ways, with typewritten directions and in my own printing. Surprisingly enough, the handwritten ones go over much better. Perhaps it's also because they know that these can be easily scrapped and written again if something isn't clear – not quite so "final" as printed ones! And it often encourages the children to try writing their own – they do a fine job, believe me, once they see what the game is! Have fun, and good luck!

Now for the work cards . . . . . (See Appendix.)

NOTES TO TEACHERS

The notes that follow are intended to be read after reading through the series of work cards which go with each topic. (See Appendix.) The set of cards, of course, is arranged in the same order as the notes, so that it is reasonably easy to locate precisely the topic one wishes to pursue.

A yellow card starts each new "project" or topic – whatever you prefer to call it, and the green ones that follow are, in each case, a more directed version of the yellow card.

These are the projects the children themselves have enjoyed working with the most, as well as the ones I have found to be most productive in the formation of concepts I feel are important ones at this level. AGAIN LET ME EMPHASIZE THAT THEY ARE NOT INTENDED TO BE A CURRICULUM for nine-, ten-, and eleven-year-olds, but just some interesting and productive ideas that do work in a mathematics laboratory situation.

OTHER NAMES FOR NUMBERS

Using adding machine tape to create long strings of "other names for numbers" has proved to be a very profitable game for all upper elementary-age children. Simple as this activity may sound, many of these older children really have very little realization that 9 + 7 is another name for 8 + 8 or 6 + 10 or 4 x 4 and 2 x 8 and 4^2, etc. They seem to look upon 9 + 7 as something to put an answer at the end of, which must be 16 and nothing else! To them 9 + 7 = 3 x 4 + 4 looks very strange indeed! Many can write beautiful trains of addition and subtraction but cannot shift over into using all four operations. How many times have your children encountered a page of equations looking like 3 x 3 = □ - 6 and filled the frame with 9? Adding machine tape works wonders to clear up this misconception! This activity will often open up a whole new world of numbers and patterns to the children who have been locked up with a single idea. Encouraging the children to use the whole roll of tape usually keeps the motivation going at top speed, and a little contest to see who can create the longest strip (without error, of course!) in a given length of time can add to the excitement IF the children involved in this race are able to cope with competition successfully. This is also an activity that can be done over and over again by the children during the year, each time expanding their concepts and computational skills. My children often work in pairs, taking turns writing and thinking up "other names."

I would suggest using magic markers for the writing, and would encourage the children to write as large as possible on the tape, for when they are finished it is fun to post the tape down the hall, around the room, wherever it can be done impressively to show the quality as well as the quantity of work it took to finish. Written with magic marker, it is visible for quite a distance! And don't worry about checking for errors – as they are spotted, they can be indicated with a notation of "I don't agree" or an arrow or a question mark. Usually the children will catch their own errors when working in pairs or groups, and can usually find a way to fix them so they work.

For the "turned-on" bunch, just give them the tape, magic markers, and the yellow card; for the more timid, suggest a small number to get them started, like 7, and the green cards. It might even help to write a couple of names to give them an idea; 7 = 1 x 7 = 15 - 8 = 49 ÷ 7 = and let them take over from there. Maybe some child would be better off working with just one operation at a time until he gains confidence. Start with subtraction, maybe, then suggest a move to division, or multiplication, etc. This is a good way to get a child to practice the operation he is weakest in, or at least with which he might need additional practice.

With card one in the green series, if your children haven't played around with negative numbers, they will very quickly reach a limit of possibility with a small number. Go on to the next card. Watch what happens, and perhaps come in with a question like: "Can you find more names with subtraction than you can with addition?" "Why?" Same with multiplication and division; knowledge of negatives extends the possibilities, and so does intuitive knowledge of fractions. Sometimes, if you step in and ask a couple of leading questions like "How many 1/2's in 2?" There is a name for 4! "How many 1/4's in 1?" Another name for 4! If they pick up the pattern, they can often continue for quite a spell!

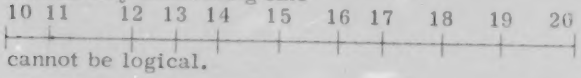
My students have really become fascinated with this game, and several have come up with some most unusual patterns in division and multiplication. Many have learned the principle of halving and doubling the factors to come up with the same products, and what happens when you add zeros to a dividend and a divisor. (There I go using mathematical vocabulary, which I am NOT denying eventually comes in handy to know, but I would not make an issue of it with the children – at this level.) If they refer to "the number I am dividing by" or the "number I am dividing into" – and use them correctly – I am happy.

And in spite of the many times I have gently tried to get the children to realize that $\frac{2}{4}$ means nothing more than 2 ÷ 4, it took two or three sessions with the adding machine tape for a number of the group to come screaming to me about a discovery! "Hey, Mrs. Charbonneau, did you know that $\frac{8}{4}$ is just another way of naming 2?" How about that!

NUMBER LINES

Watching children make number lines can give a teacher considerable insight into the thinking of the children involved. Do they start right in working from the left edge of the strip of paper, and make rather helter-skelter marks along the row, and stick on numbers in sequences as simple as ones only, or twos? How do they mark off the interval between numbers? Let them start exactly as they wish; then as you watch their progress, ask a question here and there and make a suggestion, but refrain from giving directions. What you are out to find out first, here, is their idea about a number line, what it means, what it can show, the child's basic understanding of "numberness" – to use a word that probably doesn't even exist! That's a strange phrase, but I really don't know how else to put it – it is an understanding of numbers in sequence carried out in a logical way. What the sequence is matters not at all – it can be by 7's or 1/2's or .10, but the spacing and pattern of numbers must be logical.

I do not expect precision of nine-, ten-, and eleven-year-olds: by this I mean that I do not demand that they make these number-line segments with a yardstick and ruler in hand. However, I do suggest that these tools can help if I find that the children are totally unaware of spaces (intervals) between numbers (points on the number line), and their sloppiness is creating more havoc than anything else. If I find that a child has changed the interval between numbers, I will talk to him about a space of one unit, then work on up to the idea of adding unit spaces together to reach whatever other points he might wish to name, and perhaps something like a hand's width or an orange rod or a pencil length might act as a guide for spacing, once he understands why something like



cannot be logical.

One can get a pretty fair idea of who is ready for lots of work with fractions if they start naming these kinds of points on their number lines. Those who shy off and use only whole numbers might work this way for a while. Then suggest that they might like to try something tricky and work with halves – then extend it a bit and see if they pick up the challenge. Most children will start working from left to right: fine, but encourage them NOT to start with zero or one, but somewhere else up and down the number line.

One of the most creative children I have had takes his adding machine tape, folds it in half, again in half, and again and again until he gets down to about two-inch intervals; opens it up, runs a magic marker line down the center lengthwise to make the "line," and proceeds to use his fold marks (marked over with magic marker) to name his points. Sometimes he works from left to right, always choosing an interesting and tricky interval, and sometimes he works backwards. The most fun was to watch him do one when I suggested he put in the two end points and fill in the points in between, as many as he could.

What you want to come out of this project is a real feel for numbers, for different kinds of sequences, for spacing and intervals, and a logical approach to naming fractional points, and some adventuresomeness in naming numbers; also, you hope, some discovery about such things and 4/2 being another name for 2 or 15/6 being another name for 2 1/2 – which, believe me, is a real revelation for some children despite the fact that many textbooks have told them this before grade four or five.

Refrain from placing an already made-up number line around the room – the children come to accept it like the calendar and rarely look at it; I mean really look at it and try to make something interesting and useful out of it. Then too, ready-made number lines almost always start with zero or one – no wonder children are glued to the idea that nothing exists below zero! Now if the children themselves decide to make a long number line and either leave space available or work below zero, great! Put it up, add to it from time to time – that is, encourage the children to add to it, and see what comes.

For the child who needs more guidance, use the green cards that guide him from left to right at first and talk about even spacing and logical intervals. Then when he gets quite good at this approach, go on to the work from right to left, and finally with only the end points given, but be careful what interval he ends up with. I can remember giving a child a strip of tape and telling him his two end points were 1 and 10! The child had a terrible time with it and I couldn't figure out why it was taking him so long to figure out a simple ten-space interval, when suddenly as I looked at it, I realized that it WASN'T a simple ten-space interval – it was a quite difficult nine-space one!

This is a project that can be worked on again and again throughout the year, with each segment getting progressively more difficult. The children love to keep their segments and see how much more tricky they can get each time. Once in a while it can be quite valuable to take a poorly done number line (with absolutely NO reference to who did it) and a good one (again anonymously), make overhead projector slides of them, and get a small group together to discuss them: their good points and bad ones, why, etc. My approach with the bad ones is usually "Please help me out with this one. I'm not sure what I'm supposed to be able to find out about numbers from this." And never hesitate to suggest that the wastebasket might be the best place to file some piece of work that got messed up – adding machine tape is cheap and starting over can be so much more helpful than erasing and fixing.

As the cards indicate, this is a project that can and should continue all year long, and is one that the children never seem to tire of – they can always find something interesting to weigh and an interesting way to record its weight.

Pan balances or balance scales with gram and ounce weights are the equipment needed, as well as blocks, marbles, or almost anything to balance the scales. Simple pan balances can be made by the children, and this is a great exercise in itself. Nine-, ten-, and eleven-year-olds can accept relatively good balance, so the initial pan balance used need not be perfect. The pan balance can be made from aluminum pie tins, heavy cord, a yard or meter stick, and a stand which is quite simple to make. They can experiment for hours about where to hang the pans, etc., and loads of learning about balance, and even the density of wood, its imperfections, etc., comes into play. We even started with a sandpaper balance before a pan balance. This was nothing more than a block of wood, a piece of two-by-four, sanded semi-round on the top and covered with sandpaper to reduce the friction. We used a meter-long board about six inches wide to balance on this "fulcrum," and proceeded to find out what would and would not balance and where. Needless to say, because of the crudeness and heaviness of our "balance board," we could not weigh or balance anything of a delicate nature, but our Base Ten Dienes blocks, and our Cuisenaire Prisms and Cubes, as well as shoes, bottles, books, etc., worked very nicely. One of the major concepts we all learned was whether you marked the balance arm starting from the middle out, or whether you worked from the outside in towards the middle! Hmmmmmm?! I was surprised!

Our initial try with a pan balance was not too successful either, but with lots of trial and error, we produced one that works quite well, and a balance beam for use with metal or plastic washers for just straight numerical work as well. I cannot urge you strongly enough to try making these things; it's great experience for student and teacher alike, and by no means wasted time, even if it doesn't work!

As you can see, the yellow card is very open, and leaves the whole project up to the student to carry on by himself throughout the year. The green cards start the children off with nonstandard units of weight; the first card refers to "SEE cubes." These are the most kinesthetically appealing and visually attractive plastic blocks – clear plexiglass, I guess really – I have ever seen, and I personally HATE plastics and have very little of this material in the lab. (My dislike of plastic comes from several years' experience with it. I find that it does not hold up well under the strains and stresses of child use daily. Wood or metal is much more durable.) But I do have to commend these plexiglass cubes – they are not only appealing, they work magnificently for weighing on the more sensitive balance scales, as well as the very primary pan balances.

With respect to recording with nonstandard weights or not-exact "answers," there are several ways of doing it. One, and the simplest, is to write a statement like "The orange cube weighs more than fifty marbles but less than fifty-one.", and this, of course, can be translated into mathematical language quite easily:

$$\underset{\text{marbles}}{50} < \text{the orange cube} < \underset{\text{marbles}}{51}$$

Once standard weights are used, there are many possibilites, including graphing, which is not only fun but very colorful.

I personally like the written-out statements as well as any method: such things as "One measuring cup of dried peas weighs as much as two measuring cups of rice," and "30cc of coffee weighs 14 grams less than 30cc of iron filings," but it is very important that the children learn to use all methods of recording from lengthy, well-written English to short mathematical symbols to pictorial representation, so do encourage them!

This unit can be used before any of the specific ones on area and/or volume, but just keep in mind that for some children finding lengths does not necessarily have to come first. My experience has indicated that area is sometimes easier, and occasionally volume makes more sense to a child.

Linear measure allows for some of the simplest, yet creative tools to work with: sticks of varied lengths (children on different teams or groups do not need to use the same measuring device), even tongue depressors, pieces of rope, a surveying chain, trundle wheels, steel tapes, cloth tape measure – and other things the children can find. I have given strong direction to finding out "all about our campus" on both the yellow and green cards: perhaps you are not set up to find this useful or productive or even possible. The whole set can be re-done to apply to a school building, a section of a building, the school playground, even to things in the room IF the children must be restricted.

Basically, I want children to experience different kinds of measure with different kinds of tools, and especially not to feel thwarted or as if they must "wait" if someone has just the tool they were planning to use. One of the important lessons to learn in measuring is that it is not the unit you use that is of primary importance, it's how you tell the rest of us what you did so that we understand. And, of course, it is precisely that, communication, which standardized the units we do use today. What helps with this whole project now is that wonderful chart I spoke of in the beginning section of this book which shows in words and pictures the history of measurement. When someone (or several) chooses this card, I leave him alone for several days to see how it develops, and if ideas begin to bog down, out comes the chart to look over and study.

The third green card is concerned with "how big is" – the soccer field, the football field etc. – deliberately ambiguous. What does "big" mean? What kind of measurement should or could be done here? COMMUNICATION, again – a mighty important item, and this time, on my part, for it really isn't clear what is meant. And it does seem to generate the kind of conversation and decision-making that I hope for here. The whole set of green cards progresses from linear to area to volume. The yellow card implies it, and usually it happens with the children who use it.

SCALE DRAWINGS

This is one of the most popular activities in the lab for all the children. Usually they do start out making scale drawings of real places or things, something that belongs to them and that they are very familiar with, and then branch out into designing and planning quite elaborate structures. One thing that has helped intrigue them with scale drawings is a box of blueprints donated to the lab by an architect father. This collection has been well used, and some of the more interested and adept youngsters have built their scale models from these blueprints. The biggest bonus from this unit, as well as several of the others, is the practical use of mathematics in life – over and above the "grocery store arithmetics" with which the children are usually confronted. Here are situations in architecture and building that their fathers deal with daily, and the children can really immerse themselves in practical but fascinating problem-solving. The cooperation we have received from parents is terrific; field study trips of all kinds and varieties that go way beyond the look-and-lecture kind. And fathers who will give their time to come to school and work with the students on their various building projects have been a tremendous help.

If the children are doing scale drawings of rooms or things or buildings around school, you can keep watch on the measuring they are doing, and be ready to help in situations where you are needed. (One group doing a classroom inadvertently picked up a yardstick and a meter stick to measure with, and couldn't figure out what went wrong until I silently held the two sticks side by side for them to see!) The more creative children seem to have little trouble in working out an appropriate scale for their drawing, or in choosing appropriate graph paper or drawing paper on which to work. For the children using the green series I have suggested using inch or half-inch graph paper to help with the scale. If they get into real difficulty and you can sense a lack of understanding about what "scaling" is all about, an interesting and fun exercise is to give the children a piece of inch graph paper and ask them to draw an interesting shape with a pencil and ruler. (The restriction of a ruler tends to keep the shape within easy bounds to scale down, or up!) Then give them a piece of half-inch graph paper, and a piece of two-inch graph paper and ask them to "copy" the shape, both larger and smaller.

Watch carefully! Some children are quick to realize that some kind of code system for locating specific points on their original drawing on the new sheet is highly desirable and will proceed to create a grid system similar to those used on maps, with letters up the side and number across the top or bottom. Others prefer to use a well-developed hand/eye coordination and draw that way quite successfully. And then there is the child who has a very difficult time getting started. With this one, I usually suggest that he count the squares in his original inch sheet, and cut the half-inch and two-inch sheets to have the same number of squares as the original. Then locate key points on the original and count over and down, or over and up from a corner to relocate these points on the new drawings. Once they get the idea, they can go to work and enjoy the project.

Some of these less secure children have really taken off with scaling shapes, and have done some remarkable work in this area. One, in fact, gained some amazing insight into area with his drawings of fairly simple shapes.

This is one of the few cards that I would strongly suggest should follow in sequence to the scale drawing. However, there are some few children who can do this kind of work without the preliminary build-up of scale drawing, but very few. It is not necessary that a 3-D model of their scale drawing be made immediately after doing the drawing – they may be saturated with "scaling" by that time and want to do something else in the interim. Fine! But I would encourage them, if interested, sometime to do a scale model of whatever they did their original drawing of.

Balsa wood is expensive, so a lot of careful thought must go into its use. If the children are doing a very extensive scale model, we encourage them to purchase their own! (Which is a problem in itself, buying the correct quantities, sizes, weights, lengths!) And a child who is usually pretty careless and slipshod I would encourage to try matboard or poster card first. A balsa wood structure is really great, and does involve hours of careful mathematical and other planning to be successful!

One of the most fascinating projects in this area is bridge building. First should come a careful drawing. The next step is to build the structure from drinking straws and pins – inexpensive materials to experiment with – and it doesn't take long for the children to find out a very key concept in building: a triangle is a rigid figure and a very necessary part of a structure undergoing a lot of stress! There is a statewide contest in Colorado for junior high and older children in bridge-building, with a given amount of balsa wood that may be used. The bridge is then subjected to weight until it gives, with the strongest, of course, winning a substantial prize! We have not, as yet, had any of our boys go on to entering this as they reach the right age, but maybe one of these days someone will do so.

The green card makes a suggestion that I have found very helpful with some children. Styrofoam is very easy to cut with an exacto knife. The children first cut the floor size of each room (or THE room), and make the walls of heavy construction paper which can be easily glued or taped, cutting the windows and doors as necessary. Other details can be drawn on the paper quite easily. Somehow, working around the inch-thick styrofoam floor makes building up the sides easier, and sometimes kicks off ideas not previously thought about. One little guy neatly began to lay out a "traffic flow pattern" with yarn after gazing at his model of the lower school building for some minutes!

This unit can begin in two ways: either with the cards asking the children to make a road map from school to their house, or by working with city street maps of your city. We have done it both ways, and find either approach provides interesting activities in measuring, scaling, finding distances, computing with whole numbers and fractions. In our map boxes we have city maps of Denver, Colorado Springs, Pueblo, Aspen, Estes Park – any of the rather well-known areas of the state, as well as some of the less famous cities and towns, and one of the first things the children seem to notice is the scale of miles and how it differs from map to map.

String and ruler are the only "tools" needed to plot distances, and the boys themselves have come up with this more or less now standard way of recording their work:

place	scale of miles	inches	miles
airport to capitol	1.5 mi = 1 in.	4.5 in.	?

The children working with the yellow card seem to have the innate "feel" for the important relationship of such a map to their homes: an appropriate scale of miles, graph paper that will help work it out, and a sense of space and distance, so that they are not "shocked" or confused by their figures when they do actually plot the distance while driving to or from school with their parents. And you can check their relative accuracy from a city map yourself!

Most of the children working with the green set of cards would most likely benefit from plotting some distances from a city street map first, before trying their own, but don't discourage the child who wants to try doing his own first. It's quite possible that the street map will make more sense to him after he has tried quite hard to make one!

One of the important concepts from working with maps is the use of a coordinate system for locating places without having to look all over the map for one little spot. And almost all maps use one, of some type of another. Good discussion can come out of working with various types, and discussing the merits of each. And after working with scale drawings, some of the children became quite interested in doing a large, "blown-up" edition of just their neighborhood part of the city map, which turned out quite well, indeed!

STATE MAPS

Here is a beautiful way to coordinate social studies with math! In our school we do an extensive unit on Colorado history with the nine-year-olds, and "Man, A Course of Study" the following year, with the beginnings of American history with the sixth grade elevens and twelves. The map box has such a wide collection of all types of maps, gathered for "free" from many sources – state highway maps, local historical interest maps, weather maps, railway systems and airline systems maps – almost anything you might want to know about Colorado maps! (And for "Man, A Course of Study" there are some magnificent maps of the Amboseli game preserve in Africa, as well as others pertaining to the area being studied!)

There is so much that can be learned from maps, really looking at what they show, how they show it – the children have gained quite a bit from just having the time to pore over the different kinds of maps and discussing amongst themselves what they have found out. Many of the children have commented on what fun automobile trips are after they know how to read and work with the highway maps! And again, all the equipment needed is string and a ruler. There is a tiny "trundle wheel," a little instrument that you can run over a given roadway or trail that records the number of inches you have traveled on paper, so that you can figure out the actual mileage. I have two of these in the lab – some of the children love to use them, and others prefer the string and ruler method. The little "map markers," as I call them, are not easy to use, for they have several different scales on them, land miles, nautical miles, kilometers – and I WILL NOT show the children how to use them. This they must find out for themselves – and many do. For them, it's rather like a game, to see just how something works all by themselves. ("Play with it a while and see what it does!") Some children who resist doing something all by themselves will use the simpler, more tested approach. Maybe someday, we can have children in classrooms who are not afraid to try anything – let's hope!

The yellow card children should, on their own, investigate all kinds of maps of the state – your state. All of these cards can easily be rewritten for any state in the union, without too many hints or suggestions. The green set suggests using various types of maps for their investigations, and into this set can go specific area maps that you might want them to work with. We have one that I most always encourage the children to use, and that's a trail map of the Aspen/Maroon Bells area, which is simpler to read than a "topo" sheet, but gives all the necessary information for a back-pack or mountain hike.

This unit works best for me when I use a bulletin board pinned full of envelopes from all over the world. Usually, I put the yellow card in the middle of a large cork board, with the letters all around, and intermingle the green cards with the appropriate envelopes. This gives the children somewhat of a choice of which letters and cards they wish to work with, if any, and it makes an attractive and somewhat intriguing wall board in the math lab.

Basically, this is nothing more than another approach to map reading and finding distances in scale, but for some children this has proved to be more interesting, and therefore more of a learning situation than any of the other map units. Again, one of the bonus attractions is the extensive use of geography. I am constantly amazed at the number of upper elementary children who have a very hazy understanding of city, country, state, continent, county. They have little realization or intuitive sense about where states are located in our country, or where other countries are located. This unit gives ample opportunity to really pore over maps from all over the world, and get a good idea about how mail must be transported from one part of the globe to another – especially "surface" mail.

The interesting thing that happened with one of our children when he was working with letters a few years back was that by mistake he picked a world map with a Mercator projection, and didn't realize it was different until he plotted two distances which he knew intuitively to be vastly different, yet came out the same, or close, when he measured on the map! After it was suggested to him that he look over the scale in the corner very, very carefully, he discovered he had some adjusting to do for every latitude line he was working on! I doubt that Norman will ever forget what a Mercator projection is all about!

I would very definitely precede this unit with lots of study and poring over and work with topographic sheets of your area available from the U.S. Geological Survey office at very reasonable rates. Aside from having most of the areas in Colorado, I also have a set, available from the Survey office, of the different topographic areas in the United States, like the glacial areas, deserts, sink holes, different mountainous regions (old and young mountains), different shore lines, and river drainage basins. The children seem to love to get these out and will work with them by the hour – sometimes just talking and comparing notes, and sometimes studying very carefully a specific piece of terrain they have either just visited or are planning to visit on skiing or hiking trips.

Just by carefully looking "topo" sheets over, a child can quite soon learn the pattern of colors and the symbols used, no matter what area of the country he is looking at. There is some security in knowing that contour lines are always brown! And having the experience of looking at something from an entirely different point of view is quite interesting and enlightening. I guess one of the reasons these maps are so intriguing to our children is the strong group in mountain training in our upper school. This is a group that is very much looked up to and admired by the younger ones. Most of them cannot wait until they are old enough to become a part of this activity.

Very imaginative and creative children can, from their work on the topo sheets, create one of their own, using correct coloring, symbols, and all the mathematics and logic necessary to do a remarkably good job. Some of the boys have gone even further with their work and have learned to make cross sections (constructing side views) from a topo sheet, which requires some really intricate mathematical figuring and a genuine "feel" for the area being worked on. Children working with the green set find most generally that working with an island is less difficult than with a square of land or its equivalent. The island can still allow height (contour lines), a river valley, perhaps a volcano, some sense and logic about how high it could be, reefs, many features that are found on topo sheets that they have looked at before. As I mentioned in the main part of this study, the book Boy of the Islands provides a magnificent background for this activity, and those of our students who have read it enthusiastically dive into the study of making a topographic study. There are also some very intriguing novels involving skiing and mountaineering which help to motivate the children into this kind of study.

Geology is an area of science that fascinates me, especially the topographic and cross section studies. We have had some exciting and profitable sessions in this area, the children and I.

Editor's note: The same kinds of thing can be done with coastal charts, available at the U.S. Hydrographic Office or in marine hardware stores.

TOPOGRAPHIC MAPS IN 3 DIMENSIONS

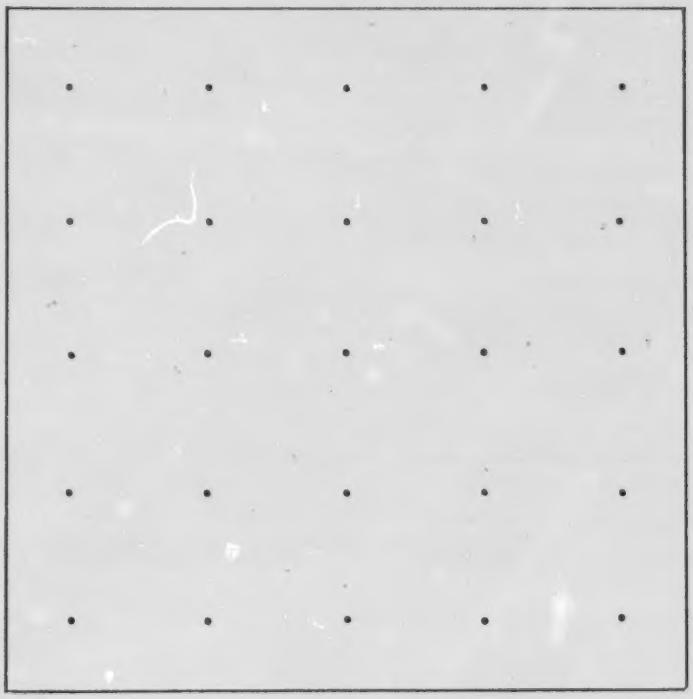
Here again, this is one place that working with topographic maps in two dimensions is a proper build-up to working in three dimensions, and I would strongly urge the children to follow their map work with the 3-D models.

The abler child with very little help and direction can proceed either to build a copy of his topographic map or to create something entirely new, or even to take a topo sheet from the map box and recreate it in 3-D. Mat board works beautifully for intricate and detailed mountainous regions, especially if the child is terribly artistic and works well with his hands. For a first try, or perhaps with the less creative or artistic types, a suggestion of corrugated cardboard or even tri-wall, which is very heavy corrugated cardboard, is a good one.

The green card is quite specific in its directions – more so than I like, but for this activity it has worked well to get the idea across, and often the children have then gone on to more intricate 3-D maps on their own. Choosing a contour interval is quite important, and again logic plays an important part in making sense when the scale is set up.

Aside from some beautiful islands in full color, we have a canyon, a hogback ridge, a volcano, and a young river basin for display and use in the lab. When the boys complete their 3-D structures, I usually ask them to write some good questions to go with them. These structures with their questions often serve as the motivation to another group of children perhaps later in the year, and perhaps next year even!

This is a 25-nail geometry board, better known as a "geoboard."

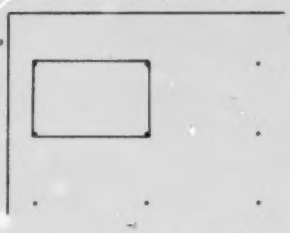


It can be easily made from 1/2-inch plywood, from a square 10" by 10" with linoleum binding nails driven in about half way, every two inches, starting an inch from the edge of the board. Heavy, colored rubberbands can then be stretched from nail to nail for all kinds of interesting work with shapes.

GEOBOARDS: FINDING AREA

Using a geoboard to approach the idea of area, and how to find area quite intuitively has been a very popular activity in the math lab. The children in the lower grades have been building simple shapes with one rubber band and talking about "how many squares are inside?", so that when they reach grade four and begin to get more adventuresome and creative with their shapes, they start to look for patterns in the relationship between their shapes and the area. And they also begin to get very interesting ways to find the areas!

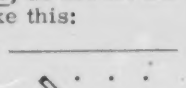
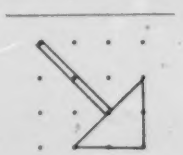
It is almost imperative to have literally thousands of two-inch squares of heavy paper cut and ready to use with the geoboards. For some children, this makes the idea of finding area almost pop out and tell the "secret"; for others, who are working at a more difficult level, it makes it possible to find out some of the generalities you want them to discover. Cut these two-inch squares out of heavy construction paper, of different colors, for lots of times different colors will clear up a seemingly difficult point that a single color will obscure.

The square unit that is worked with on a geoboard is the one encased by one rubberband, like this:  Then, when making different figures, the reference is to the number of THESE SQUARE UNITS that fit inside the shape. (This square unit is a two-inch square.) As the children become proficient at finding areas of their figures, you can progress to inch square graph paper, and talk about those units. And a fun project is then to transcribe all the previously done geoboard problems with area to inch paper and re-do them, this time in square inches instead of the two-inch square units. The very interested children will quickly discover the relationship between a unit whose sides are one inch and a square whose sides are two inches, and perhaps get going on another tangent of how squares grow.

Besides proficiency at finding area intuitively, some of the major ideas you hope to have the children come up with are the standard formulas for the area of a square or a rectangle (length times width), the area of a triangle (base times height divided by 2 – which seems to make more sense to a younger child than the commoner way of using it, 1/2 of base times the height), the area of a parallelogram (length times height or base times height) and the area of a trapezoid [1/2 of (base 1 plus base 2) times the height], which, after all, is nothing more than the areas of two triangles added together!

The yellow card is quite ambiguous and general, with broad leeway for all kinds of exploration and discovery. Keep watch on these children and ask the occasional very needed questions which will lead them to the kinds of conclusions that will help with more formal mathematics later – but not before they have made and found the areas of many, many shapes on their own. Actually, after these children have become quite good at this task, card one in the green series becomes a tougher challenge: seeing how many ways they can create shapes with one-half square inside, or with one square inside, and progress to each larger figure, always comparing for patterns. Is there a finite number of shapes that can be produced with a given number of square units? Is this related to the number of total sides of the squares involved? For instance, if you choose to work with four square units: standing alone, those four squares have sixteen sides. Is the number of shapes you can make using four square units somehow related to sixteen? Or is the area somehow related to the number of nails touched by the rubber band on the geoboard? All fun questions to play around with together with the children!

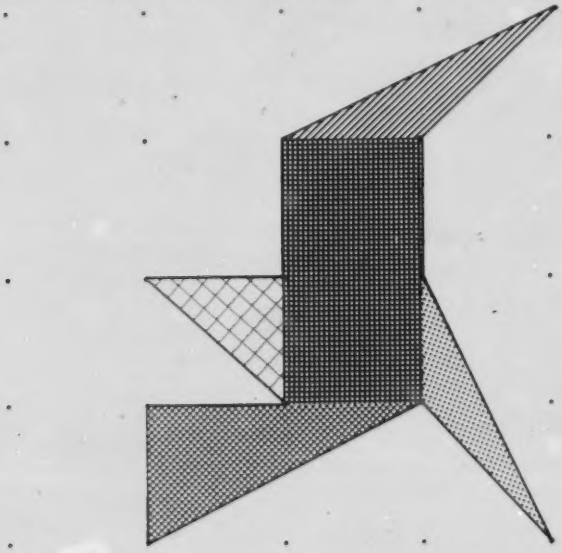
If you work with one-half square and one square, many interesting things pop out, and for us teachers, all of a sudden, many things we were exposed to in plane geometry and never understood (at least I DIDN'T!), all of a sudden become quite clear. I shall not tell you what or how – this is one of the most valuable assets in working this way – sit down and work with the children, experience some of their frustrations and excitement and joy and anger. THEY ENJOY IT AND YOU WILL TOO – and believe me, they respect you for it. There's lots I still don't know, and it's more fun to work with the children, because together you can sometimes feed each other's creativity and thereby come up with better ideas than alone! Take a geoboard home and play with it; see what you can find out about shapes and area; put one in the faculty lounge with some rubber bands and work on problems in there. You will find that others get quite interested, and then the germ spreads! The more you actually work with the materials, just as a child does, the better able you will be to understand his problems and frustrations and difficulties – far better than if I give you the answers to specific questions.

Two restrictions on area problems with geoboards: one, do not let the children cross the rubberband, like this:  or two, touch any nail twice, like this:  these kinds of figures can create problems you don't want to play around with at the moment. The first, criss-crossing a rubberband, really creates two figures, and the second creates what is called on

the geoboard " a straight line." All you have to do is tell the children that these are the ground rules for these cards. Sometime they can have totally free rein and play the game any way they want to, but for now, these are the restrictions.

The green series are the more careful structuring of the yellow card. Watch these children very carefully to be sure that they do not make shapes that are way too difficult for them. Usually they will not. They tend to stick to very simple shapes, venturing into fractions only cautiously. Let this happen until they become secure, then suggest a more tricky shape. I like the word "tricky" because it takes away the connctation that what they have been doing is "easy" and perhaps not worthy of their so-called "level." "Tricky" seems to imply fun and games, and perhaps a figure that teacher cannot do – which is always good for a laugh in my lab! AND USE THE 2-INCH SQUARES THAT YOU HAVE READY – by putting the squares directly on the geoboard shape, cutting and fitting where necessary to fit the "part-squares." The child can more easily see and eventually find for himself a method for remembering and doing these kinds of problems without the squares.

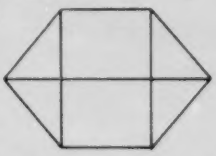
One added benefit of the paper squares came about as the direct result of one boy's ingenuity with his problem: he had made a very unusual shape on the geoboard, copied it on geoboard paper, and instead of fitting the paper squares on the geoboard itself, he had fitted the squares and part-squares, cut to fit, on his paper, and proceeded to paste them down; then he worked out the area and wrote it down. He had been quite resourceful, as well, in the way he had chosen the colors of paper squares to fill his shape: one color he used only in the whole squares that remained inside, and for each remaining corner he had chosen a different color, so that when he finished, he could better work out exactly how many squares he had used. After he had finished, he looked at his page, and said "Gee, I can do something more with this one! Each color suggests a fraction of the whole shape." And he went on to write what fraction of the whole shape each color did indeed represent!

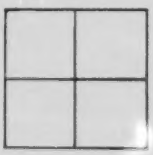


One last recommendation: ALL of these cards can be reconstructed for inch-square graph paper instead of using geoboards if you don't have any, or cannot make any. Just substitute appropriately when you rewrite the cards and proceed to ask the same kinds of questions while the children are working. Finding area on squared paper can be as much fun as with a geoboard, and as profitable mathematically – especially when you encourage the children to color their shapes and perhaps even to cut them out and repaste them on a large sheet of colored paper, or in a notebook. It's a great opportunity to coordinate art, math, and geometry.

Most of my children very quickly tire of using the geoboard paper which is the exact size of the geoboard, and have gone themselves to using the inch graph paper to record their shapes. What they do is mark the "nails" in marker pen on the graph paper, and then draw their shapes with pencil, and later in marker pen of a different color, to keep them permanent. And some have even gone to the half-inch graph paper if they want to get more shapes on a page, and particularly if they are working on the "most number of" whatever they are doing.

One group of eight-year-olds became so interested in the "fours" problem – finding all possible shapes with four square units inside – they worked in teams for days, and came up with variations on several "main" patterns. They were quite convinced that because they were dealing with four squares, each having four sides, the number of shapes they could find must have something to do with sixteen. They also got off on a tangent of comparing the area of the shape with the perimeter, and trying to find the shape which had the smallest possible perimeter. One group was convinced it was this one: while another came up with:





and proved they were right by measuring with string, as they had no experience with square roots of numbers!

This is a very specific problem for the geoboard and a very tough one! To my knowledge, there are only thirteen ways, so far, which have been devised to divide the nine square units into four equal parts – not necessarily congruent – just equal area. One of my teachers at Colorado Academy must have put at least two hundred hours into this one, and still has only eleven of the thirteen! The sixth grade became quite intrigued with it, and one team worked about one hundred and twenty hours and came up with twelve ways. One of them is still determined to find the thirteenth way, and is working off and on as he can find time and ideas.

The best way to approach it is with three rubber bands, using one to enclose the 9-square area, and the other two as "lines," stretching them from side to side, rather than trying to enclose an area within a rubberband. Be sure to emphasize that the areas must be equal, not necessarily congruent, although some solutions are. And from the work we have done, there is a definite pattern to the ways. Reflections and rotations of one solution are not allowed.

With this problem, I have asked the kids to prove each suggested solution they come up with on the inch paper, or the half-inch paper. The reason for this is there are several solutions that look so perfect, and turn out to be very, very close, but are not quite perfect – and it demands a real measure of accuracy from the children that stretches their capability quite a bit each time. This is very definitely a problem that they will want to work on for a while, forget about, and go back to as they have fresh ideas. By all means encourage this, and do not push for a finished piece of work in one week or a few concentrated sessions. Generally, we keep the solutions hidden, so that others who wish to work on the problem can start fresh and not have the advantage of previous labors – and I mean labors!

The green cards are, of course, the simpler forms of the yellow card, to get the less creative off to a successful start. Once they find they can do a task, and quite well, these children will tackle the more difficult ones with tremendous energy and stick-to-it-iveness. I often encourage these, with cards in the green series, to draw their parts on either the geoboard paper or the inch paper, color the parts, cut them out, and paste them back together to show how they fit on large sheets of colored paper; again, to coordinate art with math – and to develop a sense of pride in the children about their work. So often they will do something, slop it down on paper just because teacher insists, and then jam it away in a desk or a notebook only to get lost or mangled so badly you cannot recognize it.

The question often arises about a specific number of solutions to the tasks on the green cards. At this point, I don't know the exact number of solutions – my children have come up with varied and many ways, and we have ventured off into thinking about more ways, if we weren't restricted to the geoboard, but only used inch paper. And of course, again encourage the question about area versus perimeter; the largest perimeter, the smallest perimeter, one equal to the area, etc.

To return to the number of solutions for the yellow card – I have said thirteen, because that is the number that has actually been done by a number of talented mathematician types at the college level! Perhaps there are more, or perhaps there is a proof about thirteen. Our upper school students, to whom the lab is open at times, have become interested in programming the computer to find out. I understand that to program this problem takes considerable time and thought and experience with the electronic wizard. I, personally, am looking forward to seeing what these older students come up with!

Just one last paragraph about geoboards. Please do not feel that the few tasks I have suggested here are all that can be done with this piece of equipment. Really ar[illegible]ly, a whole book could be devoted to geob[illegible] alone. I have selected a few that can be well explored by the children themselves with very little teacher direction because I strongly feel that more is learned at this level without minute guidance or "teaching." I know one excellent junior high teacher who does all the geometry from the Addison-Wesley seventh- and eighth- grade text with geoboards, and while I haven't tried it myself, I am very sure that the geometry in the Addison-Wesley elementary series could most likely be treated in the same way. I would strongly suggest taking whatever good text you use and trying the geometry sections with geoboards to see if the concepts cannot be made much more vital, vivid, and alive.

There is another activity with area on the geoboard which I highly recommend to you as a teacher, but I wouldn't throw out to the children unless you are very sure that they have an excellent understanding of the area concept and can find the area of any shape you give them intuitively and with some structure of their own. There is a way of finding the area of any figure by the use of a "formula" or "patented method": one that is a function of the nails touched (and not touched) by the rubberband in your shape. Try this: make a shape with one rubberband that touches three nails and has no nail left untouched inside. What is the area? NOW, do this for four nails with no nails left inside and find the area. And one that touches five nails and leaves none inside? Continue until you see a pattern:

3 nails	1/2 square unit
4 nails	1 square unit
5 nails	?
6 nails	?
7 nails	?

Now see if you can think of what you can do to the first number to reach the second, like Guess My Rule or guessing functions. Then try it again, only this time, keep three pieces of data:

Number of nails touched	number of nails not touched inside the figure	area
3	?	?

and see if you can guess what rule gets you from the first number (number of nails touched) to the last number (area), somehow making use of the middle number (number of nails not touched inside the figure).

This tricky little way provides a fast way of figuring out solutions to the children's problems, but DON'T GET SO GLUED TO IT THAT YOU CANNOT FIND AREA THE INTUITIVE WAY, by adding and subtracting squares!

AREA WITH TORN OUT SHAPES

If children can find the area of strange shapes with some measuring and some estimation, it follows that they should have very little difficulty with standard shapes and the standard formulas. My experience has been that if children are permitted sufficient time to explore area in this "counting" way, with all kinds of square units, they fall into the traditional area methods very quickly and very easily. And for many children, this unit with the odd-shaped newspaper pieces is more fun and leads more easily to an understanding of area than the geoboard series. However, both approaches are good ones for a math lab.

When I start this unit with a child or a small group of children, we often begin by talking about ways to measure things; and what usually come to light are length, squares, and how much fills a given container – linear, area, and volume. If the children come up with descriptive words that cover the subject, use them – and don't worry about the proper terms at the moment. As they begin to show that they have pinned down a definition in their own minds, you might suggest that there is a standard term for "fitting squares," for instance. The outside world calls it area – so finding area means finding how many squares fit inside a shape, and sometimes those squares are inches, but sometimes other-sized squares. It is not important what size square you use, so long as you make it clear to others what that square is!

An important and fun concept to work on here is deciding what kind of square unit should be chosen for the job in question. Would it be feasible, for instance, to measure the area of the soccer field in square centimeters? Or the floor in the math lab in square miles? Choosing the appropriate unit for the task is really quite important – and some of the boys get so carried away with this idea that they will find the area of the soccer field in square yards, then convert to square inches, or square meters to square centimeters, just for fun!

You will note that I have used the word "area" on the yellow card. Most of the more talented children can cope with this easily and know what is wanted with just a brief discussion. Or, perhaps they already know. Usually, if you want to explore a word or have the children "discover" a meaning, you will find someone who already knows – great, it's just better that it comes from them than from you – it keeps the work in their hands, so to speak, rather than having all information come from teacher. If a child doesn't understand, the little preliminary discussion already mentioned, or perhaps a dictionary search would prove valuable. Play it by ear, and do what is best at that moment.

Most of your more creative children will start right out with an invented square unit – perhaps with a funny name even – and only later try the square inches and the square centimeters. If the standard unit is ignored completely, step in with the suggestion that it might be wise to include a standard measurement with all the other interesting ones!

Some of my children have discovered the "short cuts" – the "methods" for finding areas of standard figures from doing this unit rather than the geoboard ones. They will often try to tear triangles, rectangles, trapezoids, parallelograms, etc., and by actually fitting on the squares, have found these short cuts themselves! By stepping in yourself at the right time, you can ask the question that might hasten this along; with the rectangle, for instance: "How many squares fit along the length? How many along the width? How many altogether? If I tell you the length-number and the width-number, can you tell me how many fit inside? What do you need to know to find out how many fit inside? What did you do to find out? Suppose I tell you how many squares fit inside and the number measure of one side, can you tell me the other side? Supposing I give you just the number of squares inside, what can you tell me about the sides?" (THIS is a very powerful idea to explore.)

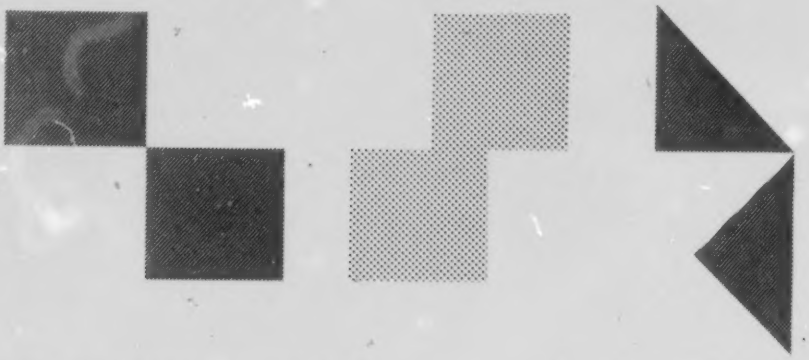
With triangles, if you can get the child to see that from any triangle, a rectangle can be built, he can easily find the area by using length times width of the rectangle, divided by two.

The green cards are programmed step by step from the made-up square unit, to the square centimeter, square inch, etc., with the hope that the child will see a pattern with these, or at least will become very secure with the concept of what area is all about. The main reason most children have such trouble remembering the standard formula for area is that they are really too hazy about the concept behind it all. Someone, or some book has said: "You find the area of a rectangle by multiplying the length times the width and expressing the answer in square units." But most of the time, no one has really taken the time to let the child explore what area is all about – to feel it, to touch it, to create it.

For some children, it would be much better to give them squares and let them make shapes with the squares first, paste them down, then get tricky by cutting some squares into triangles and making fancier shapes until this idea of area is so much a part of them that they will never forget. Then perhaps they should do the unit with inch-squared graph paper with a given number of squares, and see how many different shapes they can make with that many squares inside. And finally do odd shapes of their own creation with their own square units. Be alert to what would work best with each child, or group of children, and work accordingly. (With smaller children, like six- or seven-year-olds, it is a good start to use actual floor tiles or carpet

squares and to have them build different shapes, then draw them on large squared paper.)

On the next two pages are some very simple examples of areas made with cut-out squares. Figures 1, 2, 3, and 4 are involved with whole squares, while figures 5, 6, 7, and 8 use the same number of whole squares as the corresponding first ones, but have become somewhat "trickier" to look at as they incorporate some half-squares. You can easily see what restrictions have been used in the making of these areas – sides must touch somehow. This restricts the variety of shapes you can make, but does facilitate finding the area. If you allow a situation like:



you run into small problems with perimeter, and really, you have created two distinct areas rather than one. However, all these things are interesting, fun, and profitable to investigate with the children, so take some days and, "no holds barred," play around with these ideas! One suggestion I would make, however, is to use the two-inch squares instead of the one-inch ones, as I have here. They create a nice computational problem for the children when they are finding the area: do it first quite simply, count the squares, but then find out how many square inches there are in each shape. (Some children are very quick to say, "Twice the number of actual squares because these are two-inch squares we are using!")

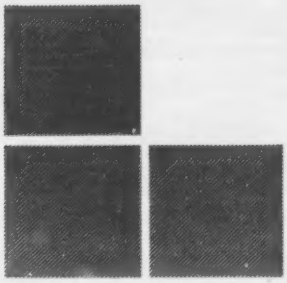
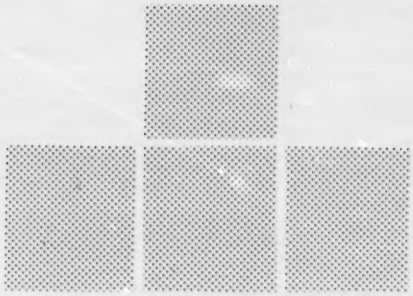


fig. 1



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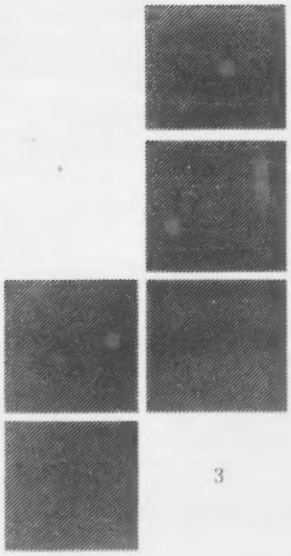

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4.

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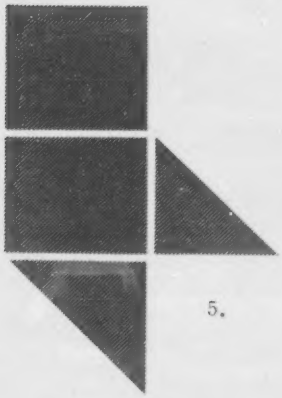

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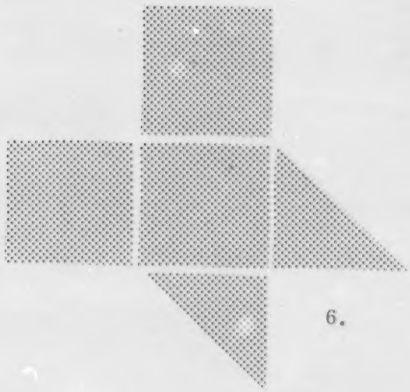

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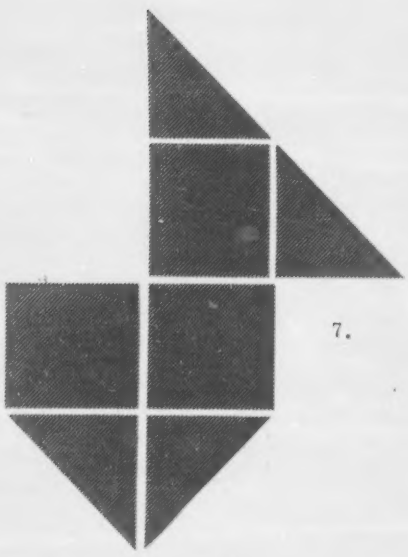

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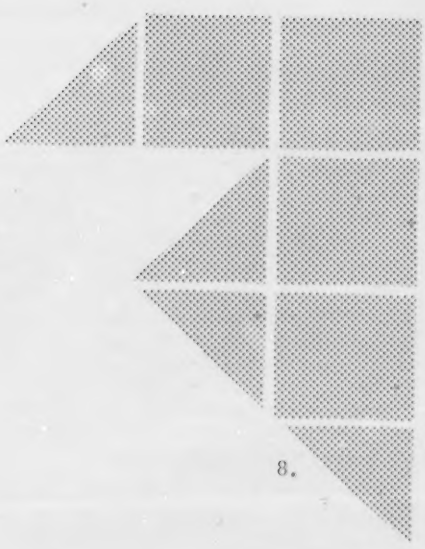


8.

This approach to finding area of nonstandard shapes can be an extension of any of the previous ones, or used in place of any others, depending on you and the children involved. Here is a situation where you can help get things going nicely, after some child has chosen to work on the yellow card, or you have tossed out the idea that it might be fun to work on this one, by setting out on the table a box of cookie cutters. They make wonderful "funny shapes" for finding area, and the children take great delight in using them and finding other "funny shapes" to use as well.

The children using the yellow card have little or no trouble estimating the part-squares that appear after they have drawn around their shapes on inch paper. Most of these kids accept quite readily the concept of estimation and can proceed to group the part-squares without getting too "hung up" with accuracy. These children seem to realize that halves, thirds, and fourths will get them where they want to go! ("A little more than half can be paired with a little less than half," etc., etc., quite nicely.) One of the ideas that seems to come out of this type of work is that there are times when one doesn't need to know exactly – an approximation will do – and also, and perhaps more important, that a good idea of estimation can help when one really wants accurate computation! For instance, if one estimates that there are about 16 square inches inside a circle, and after computing with the "formula" one gets 35.48 square inches, – oooops! better go back and check – something happened! Too many times, too many children use a given formula, get an incorrect answer, a "way-off" answer, but have no idea how "way-off" it is. – "I did what you told me. I used the right formula." Logic never enters their minds, unfortunately, and it happens to very bright children as well as to the less able ones.

Most of the abler children will also come up with rather unique and interesting ways to record the areas, too. They seem to realize intuitively that almost any measurement they make is REALLY only an estimate, depending on the accuracy of the tool they are measuring with, or their own knowledge of figures and their properties. Quite often they will say, "I know it's more than 16 square inches, but less than 18 square inches." – and quite often, they get even closer than that, especially after doing a number of them. Their way of recording has turned up in their notebooks all the way from writing a sentence such as the above, or getting quite sophisticated with mathematical notation and setting up a table such as this:

$$15 \text{ sq. in.} < \text{shape } 1 < 16 \text{ sq. in.}$$
$$7\ 1/2 \text{ sq. in.} < \text{shape } 2 < 8 \text{ sq. in.}$$

and they can prove it, too, by some unique method such as taking the shape, cutting it out, repasting the part-squares together, and counting.

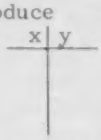
This sort of approach to problem solving is, of course, what we hope all children have – some logical, interesting, creative, unique way to proceed. But we all know it doesn't happen. Some children just don't seem to be able to think up for themselves a logical way out – this is where working with a partner often inspires the less creative child. Sometimes it even inspires him on to ways the abler one hadn't thought of. Sometimes what these children need is just a suggestion to get going. This is why YOU are needed nearby, to watch and step in at times to help, without telling exactly "how to." Your approach might be: "Gee, Andy has found an interesting way to do that – or record this – talk to him." or "Have you tried cutting out the part-squares and fitting them together to get as close to an inch square as you can, then count?" – always with the tone of a suggestion, so that the decision is still the child's. The child seems to need to experience what he does, first hand, even with lots of trial and error before success, for anything to have meaning. Second-hand experience just doesn't make it – and I am quite convinced that is why children do not remember what we think they should about the properties of figures. They really haven't "done it" – just to substitute into an equation and come up with the "right answer" doesn't give the experience needed.

The less able child also seems to be quite frustrated with the idea of a less-than-absolute answer. Estimation scares him, and I am afraid this is partly our fault, as teachers. Too often their initial experience with mathematics is one of precise answers, and if the child doesn't get it – POW – "You are wrong!" and with a big red X! Accuracy in computation is only one phase of mathematics – granted, an important one – but I tend to feel that if little children's first experiences are with estimation and logic, later with accuracy, perhaps they won't be so afraid of the idea of a good guess, or an "about" answer. David Page has a nifty booklet out called Ways to Find How Many, which is concerned first with counting, then with using logic to find larger quantities with tricky drawings of stacks of cubes. I have kept a small number of these booklets on the shelf in the math lab each year, and find them invaluable for some children in developing logical ways to find things out. Its value is certainly far greater than just working with stacks of cubes!

To work slowly and carefully with the less able child on the area of nonstandard shapes can help him tremendously in developing his own ways to solve all kinds of problems. Don't be afraid to let him make mistakes while he is exploring things and ways – then sit down and quietly discuss what went wrong and why – if he is ready. Maybe at this point, if he doesn't see why, the best thing to do is drop it, accept what he has done, and come back to it at a later date in a somewhat different manner. Or the child himself may come back to it after a time with something else that may have given him just the clue he needed in order to understand what the whole bit on area is all about.

AREA AND CIRCUMFERENCE OF CIRCLES

Finding "pi" is a discovery that most children at the upper elementary levels can make. The very able ones can find it by careful observation of what they find out and record with the yellow card. These children are usually quick to spot patterns, and, of course, are very careful to set up any data they collect in such a way as to make that pattern pop out. What has usually happened in my lab is this: the children find that there are basically three things that they can find out about circles: the distance around (circumference), the distance across the center (diameter), and the number of square units inside (area). Some become fascinated and carried away and start messing around with a circular geoboard with central angles and figures that fit inside, etc.; but for the most part, diameter, circumference, and area are the main things that catch their interest.

Way back in the primary grades we do a great deal of work with guessing patterns – call it anything you wish: Addison Wesley does a lot with this idea with its function machine, which eventually reaches the proper mathematical notation of f(x) = 2x + 3; Wirtz-Botel's Math Lab for Children, published by Encyclopedia Britannica, carries the same type of game under the title Guess My Rule, where they introduce the children to the table or with 

and the idea of finding the rule that works when, for instance 2 ⟶ 5
3 ⟶ 7
4 ⟶ 9

You can easily see this rule is 2·□ + 1 = △ or 2x + 1 = y. Other published mathematics programs for the elementary level make use of this very powerful game under different names and notations, but no matter how you set it up, it remains one of the most important games children can play, and provides them with practice and with a neat way of making use of whatever data they do collect while exploring different topics both in math and science. David Page created what he calls Number Line Games with the notation □ ⟶ □ + 7 (read as box goes to box plus 7) which uses the initial box as a starting point on a number line, and the box plus 7 (or whatever rule you wish to make) as the landing place. The landing place then becomes the new starting point, etc. If you draw the pattern of "jumps" on a number line without giving the rule, the children can then guess the rule that gives that kind of jump. Guessing Functions – a fun game no matter what you call it or whose notation you use – is a terribly useful learning situation all through school.

Anyhow, the point of inserting this here is that a table of data on diameter and circumference can help find "pi." Depending on how the child thinks, those data can be set up for division or multiplication. Division will come closer to producing 3.14 than will multiplication, but I have had children get as close as 3 1/7 with multiplication! The data are set up thus:

with the box representing the circumference, and the triangle representing the diameter. The children realize that the operation will have to be one that reduces or makes smaller the box number. Trial will show that it isn't subtraction, so the next best guess is division. Circumference divided by ? equals the diameter. And they very quickly realize it is a situation of knowing the product and one factor, so to find the other factor one divides the circumference by the diameter to get the unknown factor.

The child who discovered 3 1/7 did so by reversing the table set-up: □ equaled the diameter, and △ equaled the circumference, and then a good educated guess was made and tried about what rule would get him from the box number to the triangle number each time! The number he chose worked each time! The fun thing with this is that it can be done with both fractions and decimals, and the children can begin to see that there is really a connection between the two.

As I have said, the abler child will do this on his own. The less able one will need some guiding, either by you or by a friend who has already done it. I have tried to give some more careful clues with the green cards, but again, too much direction on any card will kill the whole thing, so keep the written directions to a minimum and be ready to step in with a clue at the right moment!

With area, it is considerably more difficult to "discover" the formula that always works. I think here I would be quite content to have the children able to come very close with good estimates. The faster children will most likely begin by asking if there isn't a definite pattern and maybe even see the beginnings of one. Suggest that pi is included in the "rule" that always works – they themselves will no doubt suspect that the diameter has something to do with it.

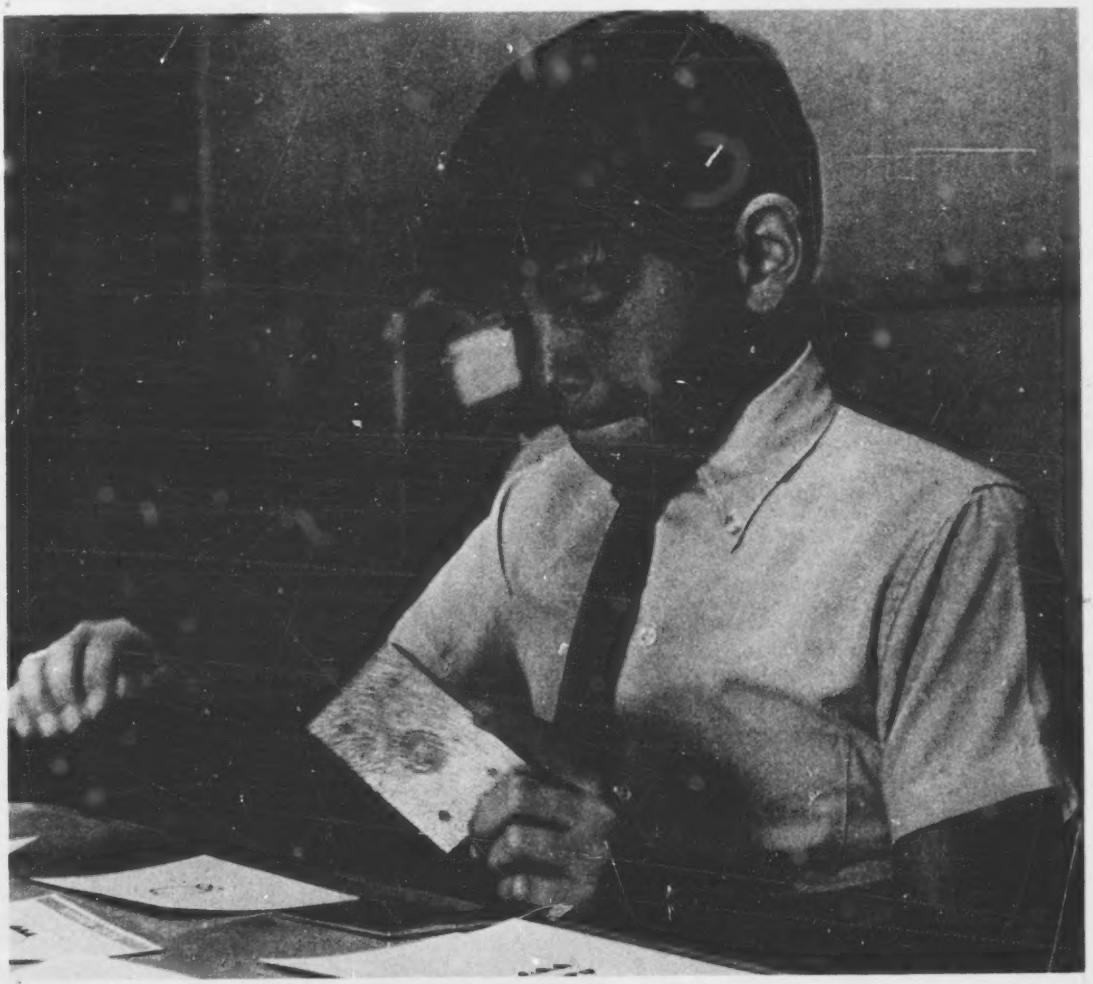
A table such as:

radius (diam. ÷ 2)	radius^2 (rxr)	area (by estimate)	area ÷ r^2

might help. If the children get part of it going, help them get the rest, or even suggest researching a math text. Getting just so far, then realizing there are places to go for help can be very revealing to the children involved, and you certainly want them to get curious enough to use established sources to nail down what they are thinking about intuitively.

For the slower child I have started with the table only after they have used the estimation approach and have a fairly stable understanding of what they are trying to find out. Then, if they are given the table to fill in with data, they too, can usually find the formula for area.

Returning to circumference for a moment, there is an elegant way to record and "find" the pattern for this mathematical investigation. The children find all sizes of circular things to work with and set up a graph with diameter readings across the bottom and circumference readings going up the side. With colored tape they fit a piece around the circumference of the circle and place it on the graph at the proper location for the diameter of the circle they are working with. The tape forms a natural graph, and the relationship between the two pops up quite pointedly!



VOLUME: KITCHEN LAB

This is a unit I would suggest as a beginning one to volume, but if it doesn't "take" well, try one of the others. My own feelings are that the others are a bit more difficult, or have shades of depth that are a bit more subtle, but I can never be sure about just what will appeal, or really work, until I've tried it! For me, this one has worked well as a beginning unit.

The kitchen lab boxes contain bottles, boxes, cartons of all kinds and shapes: all scrounged from the kitchen, carefully cleaned, and in the case of aluminum tins, their edges taped with masking tape to prevent cuts. There are standard shapes; cubes, rectangles, cylinders, cones, and some weird ones. The children themselves have brought in some fantastic containers that come to their parents, so that we really have a good collection to work with these days! Watch the waste basket and the mails -- they really help.

The most natural thing for the children to do first when working with "how much things hold," is to go to the kitchen-type measuring equipment before trying more bizarre ideas. Depending on the size of the container, they use the measuring cups (1 cup, 1/2 cup, 1/4 cup, etc.) or the measuring spoons. A fun game that has developed with the children is to estimate first with a group of containers, which holds the most, the next most, and so on down to the smallest; then see how close they were to being accurate. This has taught them a lot about careful looking at something, and using some logic and common sense about a "guess." Quite often the first time children try this they are amazed at what their first guesses produce!

With the regular containers, meaning the cubes or the rectangular solids, one of the key issues is to discover the "short way" to find volume, rather than to fill it each time. However, the children were quick to point out to me that just because, up to now, most of us think of volume in terms of cubic inches or feet or yards, it doesn't necessarily mean that this is the best, or only way. One of the boys said one day: "I can easily find out how many cubic inches of milk that gallon holds by measuring the carton, but that won't help me decide quickly how many glassfuls I can pour. I think it would make more sense with liquids to think in terms of measuring cups!" Clever guy!

With the cylinders or cones, I have been quite happy with the filling routine, unless the children have worked with the area of circles. In that case, I try to get them to find the correct "short form" or a reasonable approximation. The big thing with upper elementary-age children is that they really don't HAVE to know these things cold, verbatim, by heart, at this stage in their lives. They are going to meet them again and again in math and science, and if they leave the lower school saturated with an understanding of and a feel for finding volume, it is an easy matter to commit to memory a "formula" that one uses often. The thing to really think about is how often does the average person use a formula for finding the volume of a cone? It seems to me that if you know how to find out, this is most important, unless you work daily in a situation that requires the use of such a formula. So play it "cool" and work lots with the kinds of volume that are needed and workable, and of course, delve into the bizarre shapes because it so often helps clarify the simple, but don't feel that if "the kids don't get it now, they never will . . ." or some such, because they will, when it really means something to them.

SOLIDS: CUISENAIRE RODS

To take a good look at something and to see all kinds of different possibilities in it are qualities that I like to see developed in the children. Seeing anything from just one point of view is not only dull but dangerous! I can recall having a discussion with some Montessori teachers once about various math materials, specifically, Cuisenaire rods. The comment that was made about them by one of the teachers was this:

"Rods are a very helpful tool, but what's wrong with them is that you can do too many things with them. For a material to be efficient, it should be used for JUST one thing!" I am afraid I look at "things" very differently – they must have a multitude of uses and varieties of applicable ground rules in order to be really efficient!

This set of cards, both yellow and green, was designed with that in mind: given a "thing," a construction made out of Cuisenaire blocks (and there is nothing holy about Cuisenaire blocks – they happen to be handy, interesting sizes and colors with which to work!), what can you see in it? Size: surface area, volume, length, width, height, depth, hidden center? Shape: regular or irregular solid, faces, edges, corners (angles)? Fractions: how much of the figure is red blocks, green blocks etc.? And lots more. One of the best projects that grew from these cards was a huge, odd-shaped structure, and the question that went with it (among others) was "What is the smallest possible box that could be used for this sculpture, so that the box is not warped or bent in any way, nor is the sculpture?" And another that went with a most unusual "building": "The answer is 12. What are the questions?"

This last one generated lots of enthusiasm for finding out all one could about a particular number, like "what comes in pairs?" or threes? or fives? Great fun!

This project came out of the study on volume alone. The children had used the estimation method to try to place the containers in a row starting with the smallest (holding the least) to the largest (holding the most), and with two or three containers. They had been "way off." One of the boys remarked that it "looked like it held more because it had so much surface." – "Covering," I think, was the word he used. So we began thinking about "covering" and "skin area," as Edith Biggs sometimes refers to it!

Picking up a Morton's salt cylinder, one of the children said, "How can we find out about how many square inches would cover this thing?" One of his compatriots answered, "Easy, dummy, lay it on its side on inch paper, and roll it around once. Cut that strip out, then draw around the ends and count the squares!" Neat, what?

I must emphasize again that the green set of cards never evolves in my lab until several children or groups of children work on the yellow card. What happens to and with them decides whether some children need more careful programming – or some suggestions for details that they might otherwise overlook. And sometimes we ignore some of the green cards and pursue a tangent that comes up with one particular situation or incident. So understand that card two in the green set was created as the result of a thoughtful question on the part of one of the boys who had done considerable work from the yellow card: "I wonder what I could find out if I made some containers, all from the same size sheet of paper but with different shapes?" What he did was to produce a rectangular one, a cylinder, and a triangular one – each made from a 9" by 12" piece of heavy construction paper, fitted a bottom on with tape, and proceeded to fill each one with sand to find out which was largest, smallest, and middle-sized. THAT, of course, triggered other-shaped containers, so that we had lots to work with and to talk about.

SOLIDS AND HOW THEY GROW

There is something about doubling, the ability to double, and the ability to see how "things," numbers, you name it, grow big, which help develop a child's whole view and perspective of the world.

We do some very simple things with doubling in the lab, such as "take a number, any number, and put it in the upper right-hand corner of a sheet of graph paper and begin to double each number all the way down." It looks like this to start out with; and it grows and grows and grows. Place-value and patterns and all sorts of goodies, as well as computational practice in addition or multiplication (depending on how you do it), come popping out.

3
6
12
24
48
96
192

Another simple "how numbers grow" project is looking at squares not only growing bigger gradually, but when they are doubled. This can be done easily with Cuisenaire blocks: "Build a red square and a purple square and a brown square (all doubles of the previous color) and look at them in terms of building them with only white blocks. How many would there be in each?" Most children are very surprised to see the relationship of one to another. Or do it on inch graph paper, coloring one square, double the dimensions and color again, double again and color – maybe it is easier to see the pattern with inch paper than with blocks. Or if you are terribly abstractly oriented, look just at the pattern of square numbers. Try putting them in sets that double the dimensions each time. All of these things can be excellent build-up investigations for working with solids and how they grow.

We became involved with "bugs" and building strange creatures which we call "bugs" when some of the boys came from science one day wondering about flies and mosquitoes and bees which they had been looking at through a micro projector – huge images of the "beasts" which got the boys to thinking about how much weight the bugs' legs could support – if they were twice as big, or three times as big, or four times as big. Did the legs, for instance, have to be twice as thick, as well as twice as tall, when a bee's body was twice its original size? And what IS twice an original size? Length? Height? Weight? Proportions? So we began building – first simple shapes, and doubling them by doubling every dimension, which, we decided, was the fair way to play the game. In other words, the proportions and relationships must be kept true to the original.

The yellow card is set up for broad investigations with lots of depth; the green ones are channeled into narrower concepts, taken more slowly but with the same ideas involved. If the children can handle cubes, they can often go on into more complicated solids with little trouble.

Tessalations is the technical name for such activities, but it is such a long name, and doesn't really give a clue to what it involves, so we ignore it – and just "tile"!

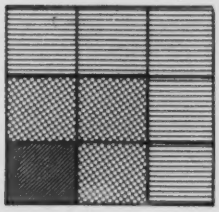
I became interested in this branch of mathematics several years ago when a colleague of mine asked, "Did you know that from squares you can build bigger squares, and from diamonds you can build bigger diamonds, and that it works with most regular figures?" Well, no, I didn't, and when I sat down to do this with some children, we found out something else quite interesting. You not only can build larger figures from smaller ones, but it follows a nice, neat progression of squared numbers, regardless of the shape of the regular figure you are using. (Regular figures: square, rectangle, certain triangles, parallelogram, trapezoid – a hexagon will not work unless you "bend" the rules a bit – and as for a pentagon, I'll leave that one for you to find out all by yourself!)

Think about the square numbers for a minute: 1, 4, 9, 16, 25, 36. Now look at the pattern of increase from one to the other – THAT is what works when you "tile" with regular polygons, and it is both surprising and interesting to the children when they find this out! We never thought about working with the hexagon until Frieda Ployer's Pattern Blocks came from EDC, and we discovered that using hexagons alone will NOT produce another hex, but by making the hex out of other pieces consistently, we could produce not only a larger hex, but with the pattern of the squared numbers!

This led some of the children to wonder about tiling with odd shapes, so we branched into this field and were surprisingly more successful than we had dared hope! Which led me to be suspicious of our carelessness in cutting, and perhaps "stretching it a bit" when we tiled – maybe the spacing in between wasn't quite accurate. One day I was staring rather intently at the cover of UICSM's Motion Geometry Book 4, with three rows of geese, beautifully tiled back and forth across the page! I was quite convinced from looking at it, as were some of the children, that we had our answer – that ANY shape could be tiled, provided it were cut with precision and tiled with precision! I made this observation this summer at luncheon one day with Roy Illsley and Michael Butler during our Integrated Day workshop – and the next day, Mike handed me a note which read, "Excursions into Mathematics," giving author and publisher and the added line "There's a beautiful explanation of your tessalations here, including the geese!" Unfortunately, I have not as yet had an opportunity to read it, but somehow, between Mike and Roy and some of the questions they asked me, I am rather sure I must be wrong.

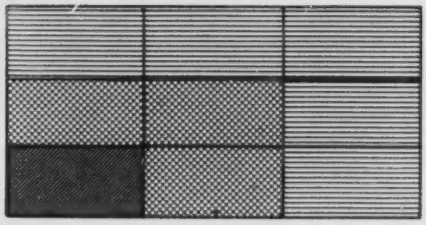
However, working on tiling with the children is still good to pursue with or without knowing EXACTLY what happens (this will be my project for the year, to find out EXACTLY what happens!), and if they discover the square number pattern and come up with some interesting ones of their own, combining shapes into interesting patterns, and find out about the properties of regular polygons and some irregular ones, I shall be pleased.

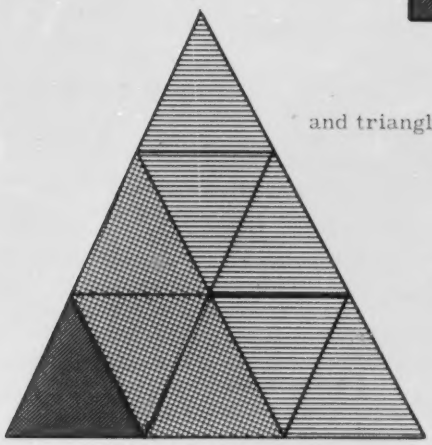
See! the pattern works for



squares,

rectangles,





and triangles! Try some of the other regular polygons!

SOFTBALL DIAMOND

This is one of a series of very practical problems that actually comes up quite often at our school, and that the students have fun solving. It sounds almost too simple – at least, I thought so, but it turned out to be more difficult than it looks.

First and foremost, no one was quite certain about how "big" a softball diamond should be. Big? It turns out we were talking about distance between bases! And the next problem to solve was the distance from the pitcher's area (no mound in softball, I have been told!) to home plate. Then there's that little problem of right angles needed to create the diamond itself. Surprisingly enough, there were some children who did not know the "diamond" was a square.

The first group that did this several years back had real trouble with the right angles. They just couldn't think of a way to make a right angle out there on the field with the cord they were using to lay out the field to be lined. It was all that two of us (science man and myself) could do to keep silent and let them struggle away and find their own solution. Finally, one of them said "How about a piece of paper – that has right angles in it!" And they proceeded to lay that down at the corners and mark it off. Most of the children now research the figures needed; then do a scale drawing, find the necessary tools with which to work outside, and lay it out. I do have, fortunately or unfortunately, a large collection of blackboard drawing tools, and usually they will go for the large protractor to get their right angles now.

This would be a great place to do some practical work with the old Egyptian method of finding right angles with the twelve-foot rope marked off in feet. Just once have I had a group use this method – one of them had just finished reading a chapter on mathematics back in those times and was dying to try some of the ideas.

This has proved to be one of the most fun and interesting of the practical problems we have worked with in the past few years. And again, it was a very real situation for the children when we first came up with this one. First Grade did need a sand pit, and we all wanted something different and as aesthetically appealing as we could make it. Now that it is in, we can find all kinds of ways to change it, improve it, or design something new and different for preschool or for our new Lower School building, plans for which are on the architect's drawing board.

To use a regular polygon, or any combination thereof, poses no particular problems for building, and it serves as a vehicle for putting into practice some of the concepts and skills which up to this point have been only "pencil and paper." Children really have very little awareness of environmental geometry: this is one way to make it very alive. I don't think children really see or understand that buildings – their homes, or school, etc. – are nothing more than basic geometric shapes with or without variations. This sand pit project has served well to this end, as well as to improve the ability to communicate ideas and directions to others, who, in many instances, will not be able to come and ask "just what do you mean by . . . !"

BACK-PACK

One of the greatest activities at our school is our mountain training program for the upper school boys in grades 10, 11, and 12. This is a well-trained, efficient group of boys whose leadership and influence on the school community are very positive. Our mountain training leader, Ernest "Tap" Tapley, is an outstanding human being as well as an excellent and superbly qualified "mountain man." The younger boys look forward eagerly to their limited exposure to him, and to the days when they will be old enough to qualify for extensive training.

As you can guess, this project is the result of plans for a weekend back-pack with the younger boys, and part of the "experience" is the planning and execution of very necessary details for such a trip. Basic packs must be considered: sleeping bags, extra clothing for each boy, as well as sharing the load for food and whatever other gear is deemed essential for a particular trip.

A tremendous amount of very real, practical mathematics and just plain "living" are in this unit, which is one that needs no prodding for motivation!

220 YARD TRACK

This also is an excursion into practical, everyday application of mathematics. And with the tools available to the children, it presents a problem with a challenge to solve.

It would be relatively easy if rope 220 yards long were available to use. All the children would need to do would be to make a rectangle with that perimeter and round the corners to accommodate the runners. No one has come up with this method yet, but I! At the moment, we have trundle wheels in yards, and we have a 100-yard steel tape that would be most applicable to the problem. And we have the soccer field, whose dimensions would help in the sense that the children could use the perimeter to decide whether the track would fit inside, on the edges, or outside, and it would help them adjust the corners to a more practical oval for running.

All in all, setting up such a track does present some nice practical problems for the children to face and solve – again giving them the opportunity to work in a very real setting, creating something that we need and will use each spring.

WEIGHT OF THE WALL

This project came about as the result of a joke. One of the eleven-year-olds was standing in the lab, leaning against the wall in such a way that I had to excuse myself each time I wanted to get by, and finally I said: "Joe, that wall really WILL stand up by itself. Could you find another place to lean on?" He looked at the wall as if seeing it for the first time, and pondered out loud: "Gee, I wonder how much that whole wall would weigh, if I could put it on a scale!" My retort was "Why don't you find out?" And he and a friend were off and running. All told, the project took them about a week to complete, as they had to look around for a cement block the right size, then find out if the wall was more than one block thick. The most interesting part of the work for them was researching the mortar and paint and working out the amounts!

They tried to find out from the library about mortar and cement: what was the difference between them; how much is used in constructing walls of a given dimension; but they were unable to find the kind of details they wanted. So they called a construction company and made an appointment to talk with someone who could really help. A little trip to the company yielded all kinds of fascinating facts about construction and kept the boys busy for days, really looking at the buildings on our campus and estimating costs, weights, numbers of bricks – you name it, they worked on it. They even became interested in the different processes for making brick as against cinder block or cement blocks.

As they worked, they involved several more boys, who, in turn, became interested in the costs and processes of building homes down in the "flatlands" around Denver as opposed to the mountains where many of our families have vacation homes. One small group carried the idea of weight on to stress, and worked on a project to determine if our lower school building could be expanded up instead of out for a new addition. This took them to the original blueprints of the building to study the foundations, and a talk with the building inspectors of Jefferson County about the "new addition." What impressed me most was the improvement in their communication, both written and oral, with adults and their own peers.

As much as our children use Cuisenaire rods throughout their lower school program, working with the metric system at the upper levels is more fun than chore. Most of them know that the rods are made on the basis of centimeters, and most have used a meter stick in some kind of measuring. These cards serve as a project to nail down some concepts and open up a new way to think about measure almost at the same time!

It is new in the sense that most of the children don't realize that there is a whole coordinated system with a simple pattern to follow or remember, really much easier than the system we use, known technically as the English system. It carries with it major benefits for future learning, as most work in the science department is done with the metric system.

The yellow card leaves broad leeway for a child to go as far as he is able or interest takes him. I think that to demand a complete set of tables in the metric system, or to push a study further than the child is apt to want to go could result in nothing more than what we see in our own system: children who can't or won't remember how many feet or inches in a yard! The metric system rather invites remembering with its pattern of prefixes;

milli-, centi-, deci- | meter or liter or gram | deca-, hecto-, kilo-;

most of the children are quite intrigued with the words alone, and will prefer the metric system to ours!

The green set does not lead them through a "logical progression" of measures of length, surface, land measure, weights, volume, capacity, etc., but rather introduces them to the idea of another measuring system, much of which they have worked with in some way, and brings up some of the vocabulary but not necessarily in order. An "out-of-place" word is stuck in now and again, in the hope of arousing interest in another branch of metric measure. If you feel this is too jarring, rewrite them to be more consistent. I do like working in consistent pattern for a while, but if I continue this too long, the children rely on it and really stop thinking about what they are doing – so the use of a discordant note once in a while helps!

CENTIGRADE AND FAHRENHEIT THERMOMETERS

This is a difficult project for most ten- and eleven-year-olds, especially if the main aim is to find the formula for translating from one to the other, and back again! However, it is a profitable one for future work in science, and lots can be gained from various lateral investigations, depending on the child. And once one "formula" is found for going from one to the other, if the child has a knowledge of inverse operations, the other is easy – the "back again" one.

The yellow card most often leads the children to books: science books, dictionaries, encyclopedia; and usually perfunctory information will head them to more interesting reading on thermometers in general. How did they originate? Who made them up? How are they made? How do they work? The question on the bottom of the card usually gives quite a surprise when the answer is found. And equating the scales on graph paper is an excellent exercise in number line work. (I have even had children say this has helped them to understand the scales on a slide rule better!)

Most all of the children can draw a thermometer and set up the appropriate scales on either side; playing "Guess My Rule" will most likely remain the province of a few adventuresome, persistent, and clever ones. However, don't hold back on trying it. Children never cease to amaze me with what they can handle when I am hesitant!

One activity with this unit which is fun is to graph the temperature hourly (outside or a beaker of water) for a week in both scales, and see if the children can find a correlation between the two this way. This is much more intuitive and lots of fun. Try it all ways and see what works best with the group you have!

The young lad who helped create and work up this project is one of the soul-satisfying treats that teachers experience all too infrequently – but when it happens, it is worth a lifetime of defeats! He came to us in grade five – a ten-year-old who was bright, intelligent, and always delightfully cheery, with creative, logical, interesting ideas to contribute – but he couldn't do arithmetic to save his neck! Not the simplest addition problem would be without an error someplace.

My approach with him was to remove him from arithmetical practice as far as possible and feed him all kinds of mathematical ideas and concepts to work on, alone and with partners (ones who were quick and sharp with skil's). This kind of combination seemed to feed on each other, and both sides benefited. My young friend gradually became able to do simple problems correctly, so that by the end of his first year with us he was beginning to have some confidence in himself in this area. And did he keep things interesting and lively with ideas and ways to do things!

In grade six, we did Stretchers and Shrinkers, which seemed to turn the tide for this boy. Suddenly arithmetic was easy and he turned his attention to finding interesting and different ways to compute. He is fabulously good with engines and cars and anything electric, and had been working with his HO gauge electric train in science. When he turned up in math lab that morning, he asked if he could work with the train and find out how fast it went, in terms of miles per hour, for each setting on his transformer. Mr. Trautman was visiting our lab that morning and worked with Scot on the project. They figured out how long it would take the train to reach Scot's house from school, five miles away, and then found the speeds.

In essence, they worked in the sequence found on the green cards, and as usual, what they did generated all kinds of interest and enthusiasm for rate problems, and soon we had gasoline-powered dune buggies, and even power boats down at the pond roaring all over, with boys timing their performances with stop watches and going on from there to find the miles per hour from feet per second!

Of course, there are some flaws in the operation, such as the problem of constant speeds around an oval of track, or wind resistance, or all kinds of important things that physics students would not and could not ignore, but for eleven-year-olds were less important. The BIG thing is their interest and enthusiasm for that type of problem, and their ability to think up ways to work it out, with and without guidance. Anyhow, the section in Stretchers and Shrinkers on rates was a mighty easy one to do after all this!

INCLINED PLANES

A small, interesting-looking package arrived on my desk one morning from Frank Trautman at the Hawken School. As usual, I didn't open it immediately, but waited to see what kind of curiosity would come from the students. It didn't take long – and what we found in the box was a haul-car and some pulleys, single, double, and triple. Somehow, the pulleys didn't engender much enthusiasm, but the haul-car created a real sensation. What could be done with it? What's it for? Why did I get it? Could they play with it a while? So play with it they did, to "see what they could find out," and what they found out was that it was great fun to run it down an inclined plane (nothing but a board a meter long propped up at one end by boxes and books), and time it with a stop watch.

It wasn't long before this pair sensed that they might produce some variance in speed by putting weights in the car and then running it down. Strange! Weights didn't speed it up or slow it down! What would? Maybe changing the height of the plane? Try it and find out! Finally, it was decided that maybe pulling the car up the plane would prove more interesting than running it down. I merely put the small spring scale on the table they were working on and continued to observe. They tried pulling it up empty, by hand, and they tried pulling it up with weights in it by hand, and while they decided it took more force with weights in it, they had no way of knowing how much more, until one of them spied the spring scale and decided they could pull it up with that! They then decided that one of the single pulleys attached to the upper end of the plane would help them keep a steady pull instead of jerking.

The results of their experiment ended in a graph, also their own idea, with the weight including the car marked across the x axis, and the amount of force it took to pull it up marked up the y axis. They tried this little experiment for several days, varying the height of the plane each day, and found a consistent pattern to their graph.

About a month later, they came roaring into the lab, waving a science book in their hands: "Look what we've found! It says that if we divide the length of the inclined plane by the height from the table, that should equal the weight of the car with weights in it divided by the force it takes to pull it up the plane! And we're going to try it!" And try it they did, and found a beautiful pattern and a delightful curve to their graph when they graphed their data table.

Needless to say, their enthusiasm and mine interested others, so it has continued to be a popular and profitable "plaything," and by keeping the data "quiet" and having the children talk over discoveries with me, we've been able to keep the surprise and discovery very real with each group that do it. One of these days we are going to get the upper school physics department, teacher and students, to come and help us go further with this one, as well as help us expand to pulleys, levers, and other simple machines! We have done some simple experiments this past year, but none of us – the children or I – feel secure enough with our findings to pass along what happened.

YOU TRY SOME THINGS AND PASS ALONG YOUR FINDINGS TO US!

APPENDIX

Reproductions of my 8 1/2" x 11" work cards follow. Since this book is printed in black and white, we are indicating the yellow cards by the word "yellow." The unmarked cards you are to imagine are green!

Remember: a yellow card starts each new "project" or topic, and the green ones that follow are, in each case, a more directed version of the yellow card.

other names for numbers (YELLOW)

Get a roll of adding machine tape. Choose a number & write as many names for that number as you can. You may use the whole roll! $5 = \frac{10}{2} = 5 \cdot 1 = \frac{1}{4} \cdot 20 =$

other names for numbers - 1

Get a roll of adding machine tape. Choose a number, like "7", & using only +, write as many names for your number as you can!

$7 = 4+3 = 8+^{-}1 = 2+5 = 9+^{-}2 = ^{-}3+10 =$

other names for numbers - 2

This time use only subtraction — & your number; write as many names as you can!

Can you find a pattern?

other names for numbers - 3

Now try your number with multiplication & division $\times$ & $\div$

Is there a pattern here too?

Number Lines (YELLOW)

Use some adding machine tape & make some number line segments. Be as tricky as you can!

Number Lines - 1

Use some adding machine tape to make a number line. Put your starting number at the left end, choose an interval (space), & put on as many points as you can.

Number Lines - 2

Now try it again, but start at the right end of your segment. Choose a new interval & work backwards as far as possible.

Number Lines - 3

Try something different! Start with numbers at either end & work towards the middle. Try one like

then get tricky!

number lines - 4

Make up a number line segment starting with zero & ending with 1. Fill in as many fractional parts as you can. Or start with 4 & end with 5 etc.

weighing & balancing (YELLOW)

Weigh as much as you can each week! Find different ways to record your work.

number lines - 5

Try a number line segment with negative (below zero) numbers! If you are not sure of what happens, talk it over with a friend!

weighing & balancing - 1

Use the pan balance to weigh lots of different objects. Balance the "objects" with marbles or the SEE cubes.

weighing & balancing - 2

Make a set of weights from plasticene or blocks or something! Weigh some objects with your weights & write up what you did.

weighing & balancing - 3

Try weighing 1 cup of lots of things (different). Use the small balance scales & the gram weights. Keep a record or graph your answers.

weighing & balancing - 4

Weigh the same cupfuls, but this time with the ounce weights. Can you figure out about how many grams to an ounce?

weighing & balancing - 5

Keep working with the scales & weights throughout the year. Find lots of things to weigh & decide which scales & weights are most appropriate for what you want to weigh. Be sure to record all your work!

measuring: all ways (YELLOW)

Using different units of measure, let's find out all we can about our campus, the buildings, the classrooms, the playing fields etc. Estimate first, then measure & see how close you came!

measuring: all ways - 1

Use the trundle wheel to measure distances around our campus. Be sure to estimate [educated guess] the distance first; then measure accurately. Record both your estimates & your measurements.

measuring: all ways - 2

We're going to put new tiles on the floor of the maths lab. How many will it take? Suppose we retile the whole building: how many should we buy?

measuring: all ways - 3

How big is the soccer field?
the football field?
the baseball diamond?
the Lacrosse field?
the gym floor?

measuring: all ways -4

Make a cubic foot. Then find out how many of these it would take to fill the maths lab, or any classroom. How about a cubic yard?
Estimate first, then measure.

scale drawings (YELLOW)

Make a scale drawing of the classroom, the school building, your room, your home - - - Something special that interests you! Lots of details, please!

scale drawings-1

Measure the maths lab very carefully: floors, walls, doors, windows, chalkboard, shelves, & cupboards. Keep good records.

scale drawings-2

Make a scale drawing from your records. Use the inch or half inch paper to help make an accurate drawing.

scale models (YELLOW)

Construct a 3-D scale model of your scale drawing. Use mat board or balsa wood.

scale models - 1

Use the measurements from your scale drawing to cut the floor from styrofoam. Construct walls (with doors & windows) from lightweight cardboard or heavy construction paper. Tape it together to make a 3 dimensional model of your drawing.

road maps (YELLOW)

Make a "road map" with good directions to your house. Don't forget the scale of miles!

road maps - 1

Write me a good, detailed description of how to get to your house from our school.

road maps-2

Record the distances you drive (from the odometer) on each street the next time you drive home from school. (Forget the small fractions if you wish.)

state maps (YELLOW)

Use some of the Colorado maps & find out all you can about our state.

Keep good records!

road maps-3

Use the inch or half inch paper to help with a scale of miles; & draw a road map showing how to get from our school to your house. Use as much information as you can to help us!

state maps-1

Use the road map of Colorado & figure lots of distances between places you know. Be sure to record the correct scale of miles, the distance in inches, then the correct mileage.

state maps-2

Use a railroad map from our state, & figure some rail distances between cities. How many miles of track does a particular railroad have in this state?

state maps-3

Find the Vail/Aspen Airways maps, & plot some air miles distances.

Don't forget the scale of miles!

state maps-4

Study the topographic sheets of our state. What kind of information can you get from this kind of map? Figure out some of the altitudes (heights) from these kinds of maps!

world maps, continents & countries (YELLOW)

Find the appropriate maps in the map box, & find out how far these letters have traveled. (You may use the tiny trundle wheel for maps, if you wish.)

world maps, continents & countries -1

Find an airlines route map & use some string & a ruler to figure out how far these airmail letters have traveled.

world maps, continents & countries -2

Use a railroad map to find out how far these 6 ¢ letters have traveled this time!

world maps, continents & countries -3

Perhaps you will need a world map or a specific continent map to find out how far some of these letters have traveled!

topographic maps (YELLOW)

Can you make a topographic map of an imaginary section of the country?

topographic maps-1

Draw the side view of an island. Now, imagine looking down on it from an airplane or helicopter, & draw this view of it!

topographic maps-2

Make a copy of your top view, but this time draw contour lines to show the elevation. Mark the scale of miles & the contour interval.

topographic maps in 3-D (YELLOW)

Make a 3-D model of your "topo sheet". Use mat board or heavier cardboard -- maybe even some tri-wall!

geoboards: finding area (YELLOW)

With 1 rubberband, make a shape on a geoboard. Draw your shape on geoboard paper & find the area in square units. Be careful not to crisscross your rubberband, nor touch any nail twice!

topographic maps in 3-D

Make a copy of your "topo sheet" of the island, & cut it out. Now, on heavy cardboard, trace the island & cut it out. Trim the pattern down to the next contour line, trace it onto cardboard -- cut -- trim the pattern -- trace -- cut, until you reach the top! Paste the parts together for a 3-D model & finish the detail work.

geoboards: finding area - 1

With 1 rubberband, can you make a shape on a geo-board that has 3 squares inside? Use the 2" squares to prove it. How many different ways can you do it? Draw each one on geoboard paper.

geoboards: finding area - 2

Choose a number between 4 & 8. Find as many ways as you can to make shapes on a geoboard with that number of square units inside.

Record your shapes on geoboard paper.

geoboards: finding area - 3

Make a shape for a partner on a geoboard. See if your friend can find the area in square units.

Take turns making shapes for each other & find the area. Use the 2" squares to prove your answers.

geoboards: 9 square "cake" (YELLOW)

Put a rubberband around 9 squares on a geoboard like this: How many ways can you divide this shape into 4 equal parts? Draw your solutions on inch graph paper to prove them!

geoboards: 9 square "cake" - 1

Put a rubberband around six squares like this:

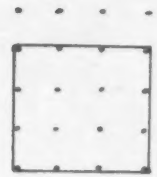
How many ways can you cut this shape into 2 equal parts? Draw the ways on geoboard paper or inch paper & color the parts.

Geoboards: 9 square "cake" - 2

Use the same shape as on the last card, but this time cut the shape in 3 equal parts. Draw the ways on inch paper & color the parts.

Geoboards: 9 square cake - 3

Start with this shape:



Cut it into 3 equal parts, in as many ways as you can. Draw your solutions on inch paper & color the parts.

Geoboards: 9 square cake - 4

Start with the same 9 squares as last time, but now cut the shape into 2 equal parts in lots of ways! Draw & color your solutions on inch paper.

area with torn out shapes (YELLOW)

Tear some funny shaped pieces of newspaper. Using different square units each time, find the area of your shapes in lots of ways.

area with torn out shapes - 2

Now see how many 2" squares fit on your shape!

area with torn out shapes - 1

Tear a funny shaped piece of newspaper. Make up a square unit of your own. Cut lots of them & see how many it takes to cover your shape.

area with torn out shapes - 3

Take your shape & this time find out how many inch squares cover it.

& now cover it with white rods! What size square is a white Cuisenaire rod? (1 face, that is!)

area of funny shapes (YELLOW)

Find all the funny shaped things you can, including your hand & foot (with & without your shoe) & draw around them on inch paper.

Estimate the area of each shape.

area of funny shapes-1

Draw around your shoe on inch paper & your foot with your shoe off. Find the square inches inside each footprint.

If you have trouble with the part squares, cut them out & fit them together.

Remember that all you can get is a good estimate!

area of funny shapes-2

Now find lots of funny shapes (try the box of cookie cutters) & do the same thing until you get good at it!

area & circumference of circles (YELLOW)

Find out everything you can about circles.

Materials that might help: string, inch paper, compass, flexible plastic ruler, circular things in the maths lab.

area & circumference of circles - 1

Measure the diameter & the circumference of as many circular things as you can find.

Set up a data table like

□	Δ

□ = { circumference or diameter

Δ = { circumference or diameter

& play "Guess the rule".

area & circumference of circles - 2

Draw lots of circles on inch paper: start with a diameter of 1", then 2" etc.

Count the squares inside.

You will have to estimate (careful guess) the part squares

Volume: Kitchen Lab (YELLOW)

Find out all you can about how much the containers in the "Kitchen lab" box hold. How many ways can you think of to record the volume of each one?

Volume: Kitchen Lab -1

How many cupfuls of sand will each of the containers in the "Kitchen lab" box hold? How else can you measure "volume"?

[how much something holds]

Volume: Kitchen lab - 2

Make a cubic inch (leave one side open) from mat board. Find out how many cubic inches of sand fill your containers (the ones you had from the first card). Can you find a quicker way to get the volume?
Be sure to record what you find out.

Use the extra Cuisenaire rods & glue together some "buildings" or "bugs". Now make up some interesting questions to go with your constructions!

Get out the extra Cuisenaire rods & build an interesting "thing". Suppose your "thing" were built out of white rods only — how many would it take? reds? greens? Keep going - - - - !

What is the "smallest possible box" your "thing" will fit into without distorting the box or your construction?

Can you make up some other questions to ask about your thing? Be as tricky as you can!

Solids & surface area (YELLOW)

Get about a dozen different "solid" objects- boxes, blocks, containers of all shapes. Compare the surface area of each with its volume.

Solids & surface area -1

Can you think of a way to use inch paper to help you find the surface area of some of the cans, bottles & cartons in the "Kitchen lab" box?

Solids & surface area -2

Can you find (or make)
some containers whose
surface area is greater
than its volume?
equal to its volume?
less than its volume?

solids & how they grow (YELLOW)

Use the box of rod "buildings" & your own rods. Pick a building. Then see if you can build, with your own blocks, a building that is twice as big but keeps the same proportions? (& shape!)

What is the volume of each? the surface area? Try this with several "buildings" & see if there is a pattern.

How about 3 times as big?

solids & how they grow - 1

The white rod is a cube.

Can you build a red cube?

a green cube?

Purple?

How many whites would make a red cube? green?

Keep going - - -

Were you surprised?

solids & how they grow - 2

Sort your cubes into sets, where the next cube in line is twice as big as its neighbor. Make a table like this & fill it in:

cube	length	width	height	if built only of whites
white	1	1	1	1
red	2	2	?	?
purple	4	?	?	?
?				

solids & how they grow - 3

Get out the box of rod buildings & your own rods. Pick a building & with your own rods, build a copy that is twice as big but keeps the same proportions. Can you predict how many white rods it will take to build your copy if you know how many it takes to build the original?

tiling problems (YELLOW)

Tiling the floor with interesting patterns is great fun! See how many different ways you can come up with- use lots of color!

tiling problems-1

Think up as many ways as you can to tile a floor with squares— & use interesting color!

tiling problems-2

Now try some tiling patterns with the other "regular" figures. Use the pattern blocks if you need hints or help!

tiling problems-3

Make up a shape & pattern that is entirely your own. Have you found any shapes that cannot be tiled at all?

softball diamond (YELLOW)

The fifth grade needs a softball diamond! Can you lay one out?

softball diamond - 1

We need a softball diamond marked off! Some questions to keep in mind:

What shape is a softball diamond?

What kinds of angles will you need to know about?

What is the distance between the pitcher's box & home plate?

between the bases?

softball diamond - 2

What is the difference between a standard baseball (hardball) diamond & a softball diamond?

sand pit: area & volume (YELLOW)

First grade wants a large sand pit. Design it & give accurate directions to the building crew.

sand pit: area & volume -1

First grade needs a large sand pit. Draw a square or rectangular one first & work out the length, width & depth.

sand pit: area & volume - 2

Now design one that combines squares & rectangles. Don't forget the measurements!

sand pit: area & volume - 3

Be daring & design one that is "free form", using curves &/or angles. If you have trouble figuring out how much earth to dig out, ask someone for help!

back-pack (YELLOW)

Tap is planning a back-pack for 15 boys. Make all the plans for food purchase, clothing, packs etc.

back-pack 1

Help Tap with plans for a pack trip: 15 boys for 3 days. Plan the food purchases first.

back-pack 2

Now plan for clothing & sleeping bags & any extra gear. Keep the packs at 25 lbs. maximum weight each.

220 yard track (YELLOW)

Lay out a 220 yard track for the runners!

220 yard track-1

When you mark off a 220 yard track for the runners, can you use the soccer field to help with length & width?

Put flags up at 50 yards & 100 yards for sprints!

weight of the wall (YELLOW)

Find the weight of one of the brick walls in the maths lab! Don't forget the paint & mortar!

weight of the wall - 1

One of the maths lab walls is made of cinder blocks. Find a cinder block the same size & weigh it. (We have a spring scale in the lab for heavy objects.)

weight of the wall - 2

Now count the bricks (or cinder blocks) in the wall. Did you check to see if the wall was one or two bricks thick? Maybe more?

weight of the wall - 3

Can you find out how much cement was used to put the wall together? What does cement weigh per cubic foot? per cubic yard?

weight of the wall - 4

How about the paint on the wall? How many gallons were used? How much does paint weigh per gallon?

weight of the wall - 5

Put your information together & figure out the total weight of the wall! (Maybe it would be fun to work out the weight of the Lower School building!)

metric system (YELLOW)

What is the metric system all about?

Is it an easy one to work with?

metric system -1

We have lots of "things" in the maths lab that are on the "metric system".

How many can you find?

metric system -2

What is a meter?

a decimetre? (or er)

a centimeter?

a millimeter?

a liter?

Try the dictionary!

metric system- 3

How many decimeters in a meter?

centimeters in a meter?

millimeters?

centimeters in a decimeter?

millimeters in a decimeter?

millimeters in a centimer?

Can you go on?

metric system-4

What about a Kilogram & a Kilometer?

How are they alike? different?

How do they fit into the metric measures charts?

centigrade & fahrenheit thermometers-1

What do you know & what more can you find out about the centigrade & fahrenheit thermometers?

centigrade & fahrenheit thermometers (YELLOW)

Find out all you can about the centigrade & fahrenheit thermometers. Can you find a way to get from one to the other? Are they ever the same?

centigrade & fahrenheit thermometers-2

Find suitable graph paper & draw a thermometer with centigrade markings on one side & fahrenheit on the other.

centigrade & fahrenheit thermometers-3

Play "guess my rule" & see if you can find a rule for getting from one scale to the other!

rates & times (YELLOW)

How fast (mph) does our HO train go at the registered points on the transformer? Work out as many speeds as you can.

rates & times -1

We want to find out how fast the HO train goes at different points marked on the transformer. Set up the track in an oval (or circle & measure the track in feet.

rates & times -2

Now find out how many "ovals" or "circles" there are in a mile!

(Remember how many feet in a mile?)

rates & times -3

How long does it take the train to go around once? (Use the stopwatch!) & be sure to mark down the transformer setting!

rates & times -4

Now if you multiplied the number of ovals (circles) in a mile by how long it takes to go around once, what would you know?

rates & times - 5

How many seconds in a minute? Try dividing your answer from the previous card by 60 — & what have you found out?

rates & times- 6

How many minutes in an hour?

What's the next step?

Do this for several "settings" on the transformer.

Can you find some short cuts?

What can you do with the haul car, spring scale & gram weights?

Work with a partner & see what you can find out.

Weigh the little haul car with the spring scale.

What does it way?

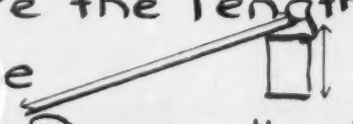
Weigh the car again with 50 grams of weight in it.

Is your answer 50 grams greater than the empty car?

Now, set up an inclined plane & pull the empty car slowly & evenly up the plane. How many grams of weight does it take to pull it up?

Were you surprised?

Start with 50 grams of weight in the car: pull it up the plane & record the force it took to pull it up. Increase the weight in the car each time by 50 grams & record what happens.

Measure the length of the plane  & the height. Divide the length by the height. Carry it out at least 2 decimal places.

$$\frac{\text{length}}{\text{height}} = \frac{\text{weight}}{\text{force}}$$

Have you seen something like this in your science book? Try it with your data & see if it works.